The Ancient Master Argument and Some Examples of Tense Logic

Fabio Corpina

University of Cagliari

Abstract

The Master Argument of Diodorus Cronus has been long debated by logicians and philosophers. During the Hellenistic period it was so famous that doxographers and commentators took for granted its notoriety and none of them gave us a detailed report. The first section presents a philosophical account of the ancient Master Argument, by trying to retrace its meaning, originated from the Megarian context, and so halfway between ancient logic and metaphysics. The second section introduces a logical analysis of the Master Argument against the backdrop of the Jarmużek-Pietruszczak semantics for the tense logic K_t4P; but the main aim of the section is to deal with one of the most fascinating attempts to peruse the Master Argument, i.e. A. Prior's reconstruction. Prior stays true to the Diodorean philosophical stance even if he uses modern logical tools. The significance of the work by Prior marks the beginning of tense logic. The third section expounds an argument by Øhrstrøm-Hasle. Danish logicians do not consider additional premises for the Master Argument. They give, *in primis*, a sentential example for the third premise, proving its inconsistency with the first two. The deterministic conclusion is the implicit result of this stratagem. Finally, in the fourth section, we compare the strategies by Prior and Øhrstrøm-Hasle.

Keywords: Arthur Prior, Diodorean logic, modalities, semantics for tense logic, time and tenses

1. The ancient Master Argument

The debate about the doctrine of potency by Aristotle is a *vexata quaestio* in the Ancient context. It had wide appeal among the contemporaries of the Stagirite and the topic was dealt with great interest by the Megarian philosophers, the strongest opponents of Aristotle.

In a first period, the Megarian thesis seemed to vouch for the position to which the *incipit* of Arist., *Metaph*. IX, 3, alludes.[1] But the more articulate Megarian thesis involves such a use of the temporal notions within the modal no-

[1] The Megarian thesis implied in this passage is made too trivial. The critics appear do not object to the official version by Aristotle. However, an alternative view which does justice to the Megarians is in Makin 1996.

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tions: the craftiest adversary of Aristotle is Diodorus Cronus with his *kurieuon logos*, i.e. the Master Argument. Several logicians evaluate the ancient Master Argument as the best rejoinder to Arist. *Int.* IX. For instance, Jarmużek 2009 introduces to a linear model within a semantics for linear future; Gaskin 1995 deals with a metaphysics of the future. It reviews the texts by Aristotle and Diodorus in light of the formalism and tools of modern logic.

The most complete report of the Master Argument is in Epict., II 19, 1. However, the report restores only the three premises of the argument and its conclusion. About the strategy we know no more than that Diodorus ruled out the third premise to obtain the conclusion. Vuillemin 1996 suggests an interesting reconstruction. It is an account in which the Diodorean argument is closely related to Arist. *Cael.* I, 283b 6-17. However we are completely unaware of what was the deduction process to obtain the main thesis: Nothing is possible which is neither true now nor ever will be. The conclusion is the consequence of the alternative view of the potency, by the Megarian Diodorus. The perspective of Diodorus links the possibility to the actuality, and it does so by the interdefinition of modal and temporal notions. In Boeth. *in Int. sec. ed., 234* is referred that Diodorus defines the possible as what is now or will be in some future instant, and the necessary as what is now and will always be hereafter.

Here is the Master Argument as it is in Epict., II 19, 1:

> The Master Argument appears to have been propounded on the strength of some such principles as the following. Since there is mutual contradiction between these three propositions, to wit:
>
> - Everything true as an event in the past is necessary
> - The impossible does not follow from the possible
> - What is not true now and never will be, is nevertheless possible
>
> Diodorus, realizing this contradiction, used the plausibility of the first two proposition to establish the principle
>
> - Nothing is possible which is neither true now nor ever will be.

In general we note that the propositions are temporally definite statements.

The first sentence means the irrevocability of the past: what has occurred in the past cannot be differently from what was the case. Therefore, it is and will always be true as a given occurrence in the past. We can state that: "If Christopher Columbus arrived in the Americas, then it is necessary that he did it"; i.e. Columbus discovered the Americas, entails that it is and will always be true that there exists a given instant in the past in which it is true that he did it.

The second sentence has often created some problems. In fact, the first step is to understand the meaning of the verb "to follow" in the context; the second step is to establish a correct interpretation of the modal notions in the tricky endeavour of avoiding a vicious circle with the conclusion.

"To follow" translates the Greek verb *akolouthein*. It has different meanings: "to occur subsequently in time", "to imply", and "to be in accordance with", are the most plausible. However the range of these meanings is very wide, and the term has a considerable importance in order to interpret the second premise. To interpret the verb *akolouthein* as "to follow in time /after" (cf. e.g. Zeller 1882, Rescher 1966), is out of place when it is used by a crafty dialectician as Diodorus.[2] From a logical point of view the most accurate translation of it seems "to infer", "to entail", maybe in a Diodorean sense (cf. e.g., Mates 1973, Denyer 1981: 40-41). But on the other hand, it would be a mistake to underrate the third solution: "to be in accordance with" hints to a kind of modal principle of non-contradiction in relation to possibility (cf. e.g., Becker 1956, Mignucci 1966: 11-15), i.e. if a proposition is possible, at the same time its impossibility is ruled out. Better yet, this formulation appears at least suggested by the wide sense of the second proposition of the Master Argument.

Another question is what "possible" and "necessary" mean according to Diodorus. The definition of the possible is obviously temporal, in the sense of "what is already now or will be in a given future". That of necessary has been usually interpreted temporally as "what is and will be hereafter", but we could also interpret it differently: e.g. "what, being true, will not be (at the same time) false"—in this case, the necessity would lose its *strictu sensu* modal value.

The fact that Christopher Columbus arrives in the Americas is not in accordance with the fact that he arrives in India. More clearly, if it is not the case that possibly Christopher Columbus arrives in the Americas and at the same time he is in India, but he is in India, then possibly he is in India. The incompatibility expressed in the previous argument is evident, because it locates Christopher Columbus in two different places. We might also say that, if it is impossible for Christopher Columbus to be in India, if the fact that he is in the Americas necessarily implies that he is in India, then it is impossible for him to be in the Americas. Therefore, if Christopher Columbus is not in India, and neither will be there in the future, this *(not-)*occurrence leads to impossibility.

2. An introduction to Prior's investigation of the Diodorean frame and his perspective on time

To look for an adequate formalisation and to guess the strategy for the ancient argument of Diodorus Cronus was an important step in Arthur N. Prior investigation of the Diodorean frame.[3] The interest for ancient logic, the debate about indeterminism and determinism, a particular attention to C.I. Lewis' modal systems, are decisive factors that stimulated Prior in formalising the Master Argument and discussing Diodorean modalities (see Prior 1955, 1967, and the critics Denyer 2009, Ciuni 2009). These researches led to the birth of tense logic.

What is tense logic? The name is in use since the Sixties of the last century,

[2] In S.E. *M.* VIII 112, the explanation for the Diodorean implication, and therefore also for the verb *akolouthein*, excludes that "to follow" may be meant on a temporal sequence.

[3] Arthur Norman Prior, born in New Zealand in 1914. He was an eclectic scholar with various interests in logic and philosophy. Prior's studies in logic are in non-classical logic area, and they have been developed in the Fifties and Sixties of the last century. But Prior was very keen on several philosophical fields (metaphysics, theology, ethics, history of logic). Prior obtained his major results in logic during his stays at Manchester University and at the Balliol College in Oxford. References are Prior 1957, Prior 1967, the posthumous collection Prior 2003. He died unexpectedly in 1969. Today a critical study on his unpublished works is underway, namely, *the Virtual Lab for Prior Studies* (http://research.prior.aau.dk/login_user.php). The *Arthur Prior Centenary Conference*, August 20-22, 2014, Balliol College, Oxford (http://conference.prior.aau.dk/) grouped together scholars from all the world.

when Prior was looking for an intermediate system between S4 and S5. His aim was to discover an adequate system that formalises Diodorean modalities.

In fact, Prior was a logician, and an expert in history of logic too. He was interested in translating modal notions into temporal ones, as it used to be in the Hellenistic period (see e.g., Boeth. *in Int. sec. ed., 234*; Cic. *Fat.* VII 13, IX 17; Alex.Aphr. *in A.Pr.* I 183,34-184,10 *etc.*), whit modern tools.

Relevant in Prior research was the investigation of the correlations between tenses and sentences.

Let us notice that Prior seems to attribute a kind of ontological supremacy to the present tense.

Prior supported a version of *presentism* (see Orilia 2012: 107-36, and Dorato 2013 for recent examinations about the theme), according to which what is present is what is real (Prior 1972: 320). Tense operators do not form propositions out of propositions: by prefixing to a sentence p a temporal operator, we specify a property in a given time. However, the sentence p alone, is already an English present progressive sentence. Using the same strategy as the ancient philosophers, Prior need not create any explicit temporal-index-link for sentences. Prior's account refers to statements which already correspond to propositional functions, and the truth-value of a proposition can vary from time to time.[4]

So, Prior's ideas are the starting point of contemporary temporal logics.

In this section we present Prior's formalisation of the Master Argument and his hypothesis about its conclusion. First, we mention Jarmużek-Pietruszczak semantics for the tense logic $\mathrm{K_t4}$ + Prior's formula $p \wedge \mathrm{G}p \rightarrow \mathrm{PG}p$, namely the semantics for the $\mathrm{K_t4P}$ system. Actually, the declared aim of Jarmużek and Pietruszczak 2009 (86) is not to express a minimal logic for Prior's Master Argument, but to study $\mathrm{K_t4P}$. $\mathrm{K_t4P}$, in fact, allows for a selection of characteristic properties. Following Jarmużek and Pietruszczak 2009, we hope to disclose the logical power involved in the Master Argument. A *strictu sensu* algebraic semantics for the Diodorean modal systems was proposed in the memorable study of (Bull 1965).

Let us briefly recap Prior's formalisation of temporal and modal operators:

$\mathrm{F}p$: "It will be the case that p" (Weak future operator)
$\mathrm{G}p$: "It will always be the case that p" (Strong future operator)
$\mathrm{P}p$: "It has been the case that p" (Weak past operator)
$\mathrm{H}p$: "It has always been the case that p" (Strong past operator)

$\Diamond p$: "Possibly p", i.e. $p \vee \mathrm{F}p$
$\Box p$: "Necessarily p", i.e. $p \wedge \mathrm{G}p$

Observe that a sentence may be true at a given time, and false at another.

[4] For instance, "It was the case that Columbus is discovering the Americas", as a sentential case for $\mathrm{P}p$, is false before he did it, and it is always true from his coming (in 1492); (2) "It will be the case that I am attending an advanced Logic course and Barack Obama is the President of the United States", as a sentential case for $\mathrm{F}(p \wedge q)$, is true from the beginning of my Ph.d career two days a week, with the proviso that Obama does not resign from his position and I will continue to work in logic. On the other hand, $\mathrm{F}(p \wedge q)$ is always false before I started my Ph.d career, and false five days a week from the beginning of my doctoral studies. Further, $\mathrm{F}(p \wedge q)$ is false both, in the case in which Obama or myself decide to leave the respective employment, and definitively false after the end of Obama's term of office.

We should think in terms of *propositional functions*: sentences are the arguments of the operators. Further, if a formula is a law, then for every substitution, we obtain a proposition true at all times.

A minimal logic for Prior's Master Argument consists of the following set of axioms and rules:

1. $G((p \to q) \wedge Gp) \to Gq$
2. $Gp \to GGp$
3. $(p \wedge Gp) \to PGp$
4. $PGp \to p$
5. $Gp \to Fp$
6. $\Box((p \to q) \wedge \Box p) \to \Box q$
7. $\Box p \to p$
8. $\Box p \to \Box\Box p$

MP. $\vdash (p \to q) \wedge p \Rightarrow \vdash q$
RG. $\vdash p \Rightarrow \vdash Gp$
R□. $\vdash p \Rightarrow \vdash \Box p$

2.1 State of the art and outline of a recent temporal semantics

Prior's study of the Master Argument focuses on the inter-definability between modal and temporal notions. Prior explicitly cites Boethius and other ancient and medieval authors, in the discussion of a *logic of futurity*.[5]

Boeth. *in Int. sec. ed., 234.* gave the following account of Diodorus Cronus' modal notions:

> Diodorus establishes to be possible, what is or will be; to be impossible, what being false, it will be not true; to be necessary, what being true, it will be not false; to be non-necessary, what is or will be false.

One of the purposes of this paper is to take advantage of the tools of contemporary logic. We aim to express some fundamental notions about time and modal categories. Boethius is the first to suggest the notions above. Boethius develops the discussion both on a philosophical and linguistic level. On the other hand, we hope to trace some developments proposed to examine a modern Diodorean system.

First, how many Diodorean systems have been examined by logicians?

Before dealing with Prior's strategy of the Master Argument—the main topic of the paper—we summarise the best attempts of building a Diodorean logic.

In fact, the Master Argument should be consistent with a Diodorean logic. Many logicians analysed different schemas for time features, both on a syntactic and semantical level.

Second, which Diodorean properties are relevant for a modal or temporal system?

Third, which class of frames does satisfy the Diodorean properties?

To begin with, it is useful to define a state of art. The next step will clarify a

[5] By admitting the rule defined like *mirror image* by C.L. Hamblin, i.e. the replacing between specular time operators, we can theorise from a "logic of futurity" by Prior, a "logic of pastness". It is sufficient to substitute P to F, H to G.

basic temporal semantics, to interpret some logics for Prior's Master Argument.

In spite of the different languages and approaches, it is possible to scientifically explain the temporal meaning of modal notions: we will show the power of a pure tense logic linked to the Master Argument. In particular, we will focus on some frames by running over the semantics for K_t4P system.[6]

It is not easy to identify the exact number of Diodorean systems. By a historical analysis, we understand that it is more a sequence of results, rather than a collection of systems.

The search for a Diodorean frame (see Ciuni 2009) starts from Prior's studies on an intermediate logical system between Lewis' modal S4 and S5.

Prior conjectured that the Diodorean Frame was an analogue of S4. In (Prior 1957) we find the author's reasons, the outstanding one was its *reflexivity* and *transitivity*. Since at the time Parry's S4.5 was believed the only intermediate system between S4 and S5 (cf. Parry 1939), later discovered to be equivalent to S5, the Diodorean frame should have been S4.[7]

According to Hintikka 1958 and Dummett and Lemmon 1959, the Diodorean frame does not correspond to S4. In fact, the Diodorean system should include the modal $\Diamond p \wedge \Diamond q \rightarrow (p \wedge q) \vee \Diamond(p \wedge \Diamond q) \vee \Diamond(q \wedge \Diamond p)$—or some analogue—to preserve a *transitive and linear* accessibility relation on the frame. However, there are some transitive frames that falsify the previous formula. So, an intermediate modal system including the axiom for linearity was gathered: S4.3.[8]

Nevertheless, Dummett and Lemmon (1959) pointed out that S4.3 does not include *discreteness*, e.g. $\Box(\Box(p\rightarrow\Box p)\rightarrow\Box \mathrm{p}) \rightarrow (\Diamond\Box \mathrm{p}\rightarrow\Box \mathrm{p})$, while we know that an adequate Diodorean system has an atomistic notion of time. So, Bull (1965) proved the Diodorean frame as *discrete*, *reflexive*, *transitive* and *linear*, and Zeman (1968) identified this logic in S4.3.1.[9]

For brevity sake, I mentioned only a schema of the most relevant results, while Ciuni (2009) provides a detailed account of the search for the Diodorean frame in a modal logic analogue system.

Many interpretations of the Diodorean system have been proposed from the Eighties of the last century. Much has been done on a semantical level (*e.g.*, White 1984, Trzęsicki 1987, or Zanardo 2009), and the Diodorean system has been interpreted in very different fields, for instance, the physical Minkowski spacetime account (see Goldblatt 1980).

I wish to take stock of the situation about some semantics of the Diodorean system in order to discuss the logic of the Master Argument. I proceed by looking into the class of frames which satisfies the Diodorean properties before comparing Prior's Master Argument to what I name Danish Master Argument. In particular, I will examine a semantics for a pure tense logical system, in the spirit of Boethius' translation from a modal to a temporal notion.

Following Jarmużek and Pietruszczak 2009, I will analyse the characteristic

[6] We obtain K_t4P system from K_t4 tense logic + (P): $p\wedge \mathrm{G}p \rightarrow \mathrm{PG}p$. Further, we know that: $K_t4 = K_t$ [1. $\mathrm{G}(p\rightarrow q) \rightarrow (\mathrm{G}p\rightarrow \mathrm{G}q)$; 2. $\mathrm{H}(p\rightarrow q) \rightarrow (\mathrm{H}p\rightarrow \mathrm{H}q)$; 3. $p\rightarrow \mathrm{HF}p$; 4. $p\rightarrow \mathrm{GP}p$] + ax. $\mathrm{PP}p\rightarrow \mathrm{P}p = K_t$ + ax. $\mathrm{H}p\rightarrow \mathrm{HH}p = K_t$ + ax. $\mathrm{P}p\rightarrow \mathrm{GP}p = K_t$ + ax. $\mathrm{FH}p\rightarrow \mathrm{H}p$.

[7] *Reflexivity* and *transitivity* are characteristic properties for S4. The above-named properties are respectively described by the following axioms:

T. $\Box p\rightarrow p$; 4. $\Box p\rightarrow\Box\Box p$.

[8] S4.3 = T + 4 + ax. $\Box(\Box p\rightarrow\Box q) \vee \Box(\Box q\rightarrow\Box p)$.

[9] S4.3.1 = S4.3 + ax. $\Box(\Box(p\rightarrow\Box p)\rightarrow\Box p) \rightarrow (\neg\Box\neg\Box p\rightarrow\Box p)$.

formula (P) for the tense logic K_t4P. Jarmużek and Pietruszczak 2009 maintains K_t4P as the pure tense logic analogue of the Diodorean system, therefore, including the fundamental premises of the Master Argument.

The characteristic formula (P), namely $p \wedge Gp \rightarrow PGp$, is the equivalent formula of what I will call (+d) in the next section, namely the second additional premise of Prior's Master Argument.

Let me first recapitulate some ideas from Jarmużek and Pietruszczak 2009.

A frame F is a (P)-frame iff the formula (P) is valid in F.

F = $<T, R>$ is defined from the relation ' $\lessdot$ ' of *immediate-precedence/succession*;

F is a LIP-frame[10] *iff*

$\forall_{x \in T}(x \text{ not-}R\, x \Rightarrow \exists_{y \in T}\, y \lessdot x)$;

F is a BC-frame[11] *iff*

$\forall_{x, y, z \in T}(x \lessdot y \;\&\; x\, R\, z \;\&\; y \neq z \Rightarrow y\, R\, z)$.

LIP-BC-frames are the class of frames satisfying LIP and BC properties. Therefore in Jarmużek and Pietruszczak 2009 (98) we find the following theorem:

THEOREM: (i) F *is a* (P)*-frame iff* F *is a* LIP-BC*-frame*.
(ii) F *is an irreflexive* (P)*-frame iff* F *is an* IP-BC*-frame*.[12]

1) Some treelike IP-frames are not BC-frames, therefore they are not (P)-frames.
2) Some linear and BC-frames are not IP-frames, therefore they are not (P)-frames.
3) Some irreflexive, transitive, right-total (P)-frames (so also IP-BC-frames) are not treelike frames.
4) There is a frame F = $<T, R>$ such that F is a treelike (P)-frame, namely a IP-BC-frame, but it is not right-total, i.e.: $\exists_{x, y, z \in T}(z\, R\, x \;\&\; z\, R\, y \;\&\; x \neq z \;\&\; x \textit{ not-}R\; y \;\&\; y \textit{ not-}Rx)$. Therefore, the *branching condition* is weaker than *linearity*. There are some branching but not-linear frames.

It may be useful to see Jarmużek and Pietruszczak 2009, in order to study and compare the resulting tree-graphics. Clearly, frames associated to *Diodorean conditions* guarantee several interpretations of the Diodorean temporal account.[13]

So far, we proposed some hints on the power of tense logics, in particular for K_t4P system. In the next step will confine our discussion to a formal strategy to explain the ancient Master Argument in the modern language of tense logics.

[10] Every LIP-frame is characterised by the relation of *limited immediate precedence* between two ordered temporal points. (i) All reflexive frames are LIP-frames; (ii) All irreflexive LIP-frames are IP-frames and conversely; (iii) All IP-frames are left-discrete and cannot have a minimum.

[11] Every BC-frame is characterised by the *branching condition*. (i) All reflexive frames are BC-frames; (ii) All right-total frames are BC-frames.

[12] Proof of the theorem is in Jarmużek and Pietruszczak 2009: 98.

[13] White (1984) considers semantical assumptions for discreteness secondary: only the assumption on irreflexivity is necessary. Differently, Trzęsicki (1987) needs a tense-logical semantics satisfying the condition of discreteness. Therefore, the author conclusion is that: even if we introduce irreflexivity, this property is not sufficient to infer the Master Argument conclusion.

2.2 Prior's strategy

Let us start with Prior formalisation of the Master Argument (see Prior 1955: 209-13).

(a) When anything has been the case, it cannot not have been the case:
$\mathrm{P}p \rightarrow \neg\Diamond\neg\mathrm{P}p$

(b) If anything is impossible, then anything that necessarily implies it is impossible:
$\neg\Diamond q \rightarrow (\Box(p \rightarrow q) \rightarrow \neg\Diamond p)$

(+c) When anything is the case, it has always been the case that it will be the case:
$p \rightarrow \mathrm{HF}p$ / or directly $\Box(p \rightarrow \mathrm{HF}p)$[14]

(+d) When anything neither is nor will be the case, it has been the case that it will not be the case:
$(\neg p \wedge \neg\mathrm{F}p) \rightarrow \mathrm{P}\neg\mathrm{F}p$

(z) What neither is nor will be true, is not possible.
$(\neg p \wedge \neg\mathrm{F}p) \rightarrow \neg\Diamond p$

The argument is a modern formulation of the ancient *kurieuon logos*. But (a) and (b) are modern translations of Diodorus' first and second premises, while (+c) and (+d) are premises from Prior. However, both (+c) and (+d) would denote two theses of Diodorus, even if omitted by his *kurieuon logos*. Then, (+c) and (+d) are introduced since they can be considered impliedly accepted by Diodorus.

In particular, (+c) is connected to the position expressed e.g. in Cic. *Fat.,* XII 27, which means: if p is true now, then at any instant in the past it was the case to say that p will be true. For, the actual now was a time in the future, seen from the past.

Further, (+d) alone does not allow for determinism. In fact, we are able to obtain, e.g., an IP-BC-frame which is branching but not-linear. Of course, we cannot say that an IP-BC-frame was in Diodorus' mind, and surely (+d) is necessary to infer the deterministic conclusion of the Master Argument.

In any case, we assume time as a discrete sequence, in order to respect a historically faithfully Diodorean interpretation.[15]

Here is Prior's strategy to prove the conclusion (z):

1. $(p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$ [*Instance of the law of transitivity*]
2. $(p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r))$ [*Instance of the law of exchange*]
3. $\mathrm{P}\neg\mathrm{F}p \rightarrow \neg\Diamond\neg\mathrm{P}\neg\mathrm{F}p$
   (a) $p/\neg\mathrm{F}p$ [*Substitution in (a)*]
4. $\mathrm{P}\neg\mathrm{F}p \rightarrow \neg\Diamond\mathrm{HF}p$
   by df. $\mathrm{H}= \neg\mathrm{P}\neg$ [*3 defined by H*]

[14] Prior considered the string $\Box(p \rightarrow \mathrm{HF}p)$ as (+c) in (Prior 1967); while in Prior 1955 (211) (+c) is not prefixed by the box ($\Box$), although the previous formulation is deduced at a later stage. In this paper we note the passage at line 10.

[15] Cf., both, *S.E. M. 10, 119-120 and previous*, for an historical view on Diodorus Hellenistic atomistic account; and Zeman 1968 for a contemporary system, namely S4.3.1, as the adequate atomistic outline for Diodorus account.

5. $((\neg p\wedge\neg Fp) \rightarrow P\neg Fp) \rightarrow (((P\neg Fp \rightarrow \neg\Diamond HFp) \rightarrow ((\neg p\wedge\neg Fp) \rightarrow\neg\Diamond HFp))$
(1) $p/\neg p \wedge \neg Fp$; $q/P\neg Fp$; $r/\neg\Diamond HFp$
[*Substitutions in (1) in order to obtain some instance of the law of transitivity composed by (+d)* $\rightarrow$ *(4* $\rightarrow$ $((\neg p\wedge\neg Fp) \rightarrow \neg\Diamond HFp))$]

6. $(\neg p\wedge\neg Fp) \rightarrow \neg\Diamond HFp$
(+d) $\rightarrow$ (4 $\rightarrow$ 6)
[*(+d) is supposed true like a premise of the Master Argument, 4 is proved, and since we are considering some instance of a law like in 5, then it is impossible for 6 to be false. Therefore 6 is proved.*]

7. $((\neg p\wedge\neg Fp) \rightarrow \neg\Diamond HFp) \rightarrow ((\neg\Diamond HFp \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)) \rightarrow ((\neg p\wedge\neg Fp) \rightarrow$
$\rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)))$
(1) $p/\neg p\wedge\neg Fp$; $q/\neg\Diamond HFp$; $r/\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p$
[*Substitutions in (1) by obtaining some instance of the law of transitivity*]

8. $\neg\Diamond HFp \rightarrow (\Box(p\rightarrow HFp) \rightarrow \neg\Diamond p)$
(b) q/HFp [*Substitution in (b)*]

9. $(\neg p\wedge\neg Fp) \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)$
6 $\rightarrow$ (8 $\rightarrow$ 9)
[*6 is proved, 8 is proved, and since we are considering an instance of a law like in 7, then it is impossible for 9 to be false. Therefore 9 is proved.*]

10. $\Box(p \rightarrow HFp)$
(+c) by RL [*By applying the necessitation rule to (+c)*]

11. $(\neg p\wedge\neg Fp) \rightarrow (\Box(p \rightarrow HFp) \rightarrow \neg\Diamond p)) \rightarrow (\Box(p \rightarrow HFp) \rightarrow ((\neg p\wedge\neg Fp) \rightarrow \neg\Diamond p))$
(2) $p/\neg p\wedge\neg Fp$; $q/\Box(p \rightarrow HFp)$; $r/\neg\Diamond p$
[*Substitutions in (2), to obtain an instance of the law of exchange composed by 9* $\rightarrow$ *(10* $\rightarrow$ *(z))*]

(z) $(\neg p\wedge\neg Fp) \rightarrow \neg\Diamond p$
[*9 is proved, 10 is proved, and since we are considering some instance of a law like in 11, then it is impossible for (z) to be false. Therefore (z), i.e. the conclusion of Prior's Master Argument, is proved.*]

Prior's proof of (z) uses (a) and (b) of the ancient Master Argument, and by adding (+c) and (+d) attains Diodorus' conclusion.

Nevertheless, Prior accepts the validity of Diodorus argument, but objects to its soundness. In fact, Prior is afraid of validating the determinism.

Prior criticizes the truth of (+d). He supposes ½ as the truth value for future propositions that are not true from now. In fact, Prior shows the conclusion of the argument, but the truth value of (z) is ½.[16]

However, Diodorus supported determinism. He would not admit a third value. Moreover, it is possible to obtain (+d) from the fourth axiom of Hamblin's system, namely $p\vee Pp \leftrightarrow \neg F\neg Pp$.[17] This is relevant since even if Hamblin

[16] During a first period, swayed by Łukasiewicz, Prior was inclined to think that the only chance to attain an indeterministic tense logic was via a three valued system. Prior (1966) seems to accept other solutions also.

[17] H4. $p\vee Pp \leftrightarrow \neg F\neg Pp$
$\neg F\neg Pp \rightarrow p\vee Pp$
$\neg F\neg Pp \rightarrow \neg(\neg p\wedge\neg Pp)$ [by *De Morgan*]
$\neg p\wedge\neg Pp \rightarrow F\neg Pp$ [by *contrapositon*]
i.e. (+d): $\neg p\wedge\neg Fp \rightarrow P\neg Fp$ [by *mirror image*]

claimed that time was dense, Prior noted that the previous axiom supports the discreteness of time, and Diodorus' account was atomistic (Denyer 1981: 49).

Further, if we contrapose (+d), we get the formula $HFp \to p \vee Fp$, that seems to codify both discreteness and determinism.

3. The Danish Master Argument

A fascinating reconstruction of the Master Argument is by P. Øhrstrøm and P. Hasle.[18]

They propose do not add any additional premise to Diodorus argument. Nevertheless, the authors require some background assumptions:

(a) time is discrete;[19]
(b) the relation $\boldsymbol{T}(t, p)$ means "p is true at t". Further, the verb *akolouthein* in the second premise refers to Diodorean implication, defined by $(p \Rightarrow q)$ iff $(\forall t)(\boldsymbol{T}(t, p) \to \boldsymbol{T}(t, q))$;
(i) the Master Argument refers to statements which correspond to propositional functions.

These assumptions should be considered along with the following definitions of possibility and necessity from Boeth., *in Int. sec. ed., 234*: $\Diamond p \leftrightarrow p \vee Fp$, $\Box p \leftrightarrow p \wedge Gp$.

The first premise of the Master Argument is as in Prior: $Pp \to \Box Pp$.

The second premise entails the concept of Diodorean implication, which is formalised as: $((p \Rightarrow q) \wedge \Diamond p) \to \Diamond q$.

Finally, the third premise is $\neg q \wedge \neg Fq \wedge \Diamond q$.

Øhrstrøm and Hasle use semantical methods to show the contradiction between the third premise and the previous two.

As a first step, they assume as a hypothesis about the meaning of q, allowing that $\neg q \wedge \neg Fq \wedge \Diamond q$, i.e. the excluded juncture by Diodorus.

"Dion is here" is q. Further, let w be the statement "The prophet says: Dion will never be here", that is supposed to be true only in the atomic instant immediately before the present instant.

Hence, Pw is false at any past time, and it is true from now on.

Prior gives an analysis of Hamblin's system in (Prior 1967: 45-50); Hamblin deals with the theme in the correspondence preserved in *Prior's Nachlass*.

[18] Peter Øhrstrøm (Aalborg University) deals with the concept of time, philosophical logic and ethics. Per Hasle (Copenhagen University) is an expert in temporal logics and computer science. They are leading the research about Arthur N. Prior and the Foundations of Temporal Logic. Øhrstrøm and Hasle 1995 (23-28) is the relevant publication to understand their philosophical background and to define what we name Danish Master Argument.

[19] In the case (a) is brought into question—but we believe it is not the Diodorean case—Øhrstrøm and Hasle (1995) suggests to substitute (a) by (A). Namely, no proposition has a first instant of truth. If a proposition is true, it has already been true for some time (Arist. *Phys.,* 236a 12-14): it is true over intervals with last but without first instant of time.

From the first premise $\mathrm{P}p \rightarrow \Box\mathrm{P}p$, we are able to get for the present time the formula $\mathrm{P}w \rightarrow \Box\mathrm{P}w$, and write the consequent as $\neg\Diamond\neg\mathrm{P}w$.

Then, we are also able to get the following matrix, by (a), where t_0 stands for the present time, t with positive n for the future, and t with negative n for the past:

t_{-3}	t_{-2}	t_{-1}	t_0 (*now*)	t_1	t_2	t_3
$?q$	$?q$	$?q$	$\neg q$	$\neg q$	$\neg q$	$\neg q$
$\neg w$	$\neg w$	w	$\neg w$	$\neg w$	$\neg w$	$\neg w$
$\neg\mathrm{P}w$	$\neg\mathrm{P}w$	$\neg\mathrm{P}w$	$\mathrm{P}w$	$\mathrm{P}w$	$\mathrm{P}w$	$\mathrm{P}w$

We deduce the *Diodorean implication* between q and $\neg\mathrm{P}w$, that is $q \Rightarrow \neg\mathrm{P}w$. In fact, it is evident that $(\forall t)\ \neg\ (\boldsymbol{T}(t, q) \wedge \boldsymbol{T}(t, \mathrm{P}w))$, therefore $(\forall t)\ (\boldsymbol{T}(t, q) \rightarrow \boldsymbol{F}(t, \mathrm{P}w))$.

From the Master Argument, and assumption (i), we get the second premise $(p \Rightarrow q) \wedge \Diamond p \rightarrow \Diamond q$, from substitutions p/q, $q/\neg\mathrm{P}w$.

Therefore we obtain $(q \Rightarrow \neg\mathrm{P}w) \wedge \Diamond q \rightarrow \Diamond\neg\mathrm{P}w$.

But by the substitutions in the first premise, we already get $\neg\Diamond\neg\mathrm{P}w$. A contradiction with the last sentence and the consequent $\Diamond\neg\mathrm{P}w$. Moreover, we also obtain the negation of the second sentence of the Master Argument, i.e. the impossible does not follow from the possible, therefore Øhrstrøm and Hasle rule out the third proposition $\neg q \wedge \neg\mathrm{F}q \wedge \Diamond q$.

Conclusion

In the second and in the third section we presented Prior's Master Argument and Danish Master Argument, respectively.

We close the paper with a comparison between these accounts; finally, we introduce a "philosophical provocation" about temporal schemas in *computer science*.

We should notice that Prior's Master Argument includes the formalisation of the original *kurieuon logos* and from four premises deduces the conclusion. On the other hand, the Danish Master Argument formalises the Hellenistic argument, but does not propose any decisive strategy to infer the conclusion. In fact, Øhrstrøm and Hasle assume the third premise, which contradicts the first two.

Moreover, Prior has used four premises, two of them are Diodorean, while the other two are supposed to be consistent with Diodorus' doctrine.

Øhrstrøm and Hasle' Master Argument achievement consists in avoiding new premises.

However, to reach their goal they require some assumptions, namely (a), (b), (i).[20]

Let's see how the premises are used in these different accounts.

[20] (i) guarantees the opportunity to substitute sentences or constants to the variables in the premise.

- The Danish first premise is (a) in Prior. It suffices to define the box (□) by the diamond (◊) and *vice versa*.
- The second premise of the Master Argumet is different in Prior's argument and in the Danish one. In Prior it is $\neg\Diamond q \rightarrow (\Box(p\rightarrow q) \rightarrow \neg\Diamond p)$, in Øhrstrøm and Hasle's version is $((p \Rightarrow q) \wedge \Diamond p) \rightarrow \Diamond q$. The second premise of the Danish Master Argument requires some interpretation for the big arrow. That is a semantics for the Diodorean connective ⇒ : $(p \Rightarrow q)$ iff $(\forall t)(\boldsymbol{T}(t, p) \rightarrow \boldsymbol{T}(t, q))$. And yet, it is provable that the Prior second premise is equivalent to the Danish Master Argument second premise.[21]
  In any case the formalisation of Øhrstrøm and Hasle extends the system from propositional logic to first order logic.
- If (A) is, in some way, valid in Prior's account also, (i) is properly present. If we substitute an actual sentence to a variable in a law we still have a tautology.
- Contrarily to Øhrstrøm and Hasle, even if Prior uses some additional premises, he refuses the assumption of the modal definitions. In fact, the same Alex. Aphr. *in APr.* I, 184, 5 mentioned that the *kurieuon logos* was proposed by Diodorus to obtain the modal definitions, in particular for the possible.

In general, Prior strategy achieves his goal, step by step, on the syntactic side via a Hilbert style proof. On the other hand, Danish Master Argument seems more perspicuous on the semantics side, by exemplifying or considering explicit counterexamples.

In both proofs, we are trying to define time and modality, the metaphysical topic of Diodorus Cronus (cf. Denyer 1999), using the tools of modern tense logic.

Let us conclude by observing that temporal logical tools can prove successful in fields as diverse as the analysis of an ancient metaphysical text and algorithm design, in particular *artificial intelligence*, *software engineering* (Galton 1987), and *model checking* (Clarke and Glundberg 1999).[22]

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[21] We prove the equivalence from Prior's formula to the Øhrstrøm and Hasle's one:

1. $\neg\Diamond q \rightarrow (\Box(p\rightarrow q) \rightarrow \neg\Diamond p)$
2. $\neg (\Box(p\rightarrow q) \rightarrow \neg\Diamond p) \rightarrow \neg\neg\Diamond q$ [by *contraposition*]
3. $\neg (\Box(p\rightarrow q) \rightarrow \neg\Diamond p) \rightarrow \Diamond q$ [by eliminating the *double negation*]
4. $(\Box(p\rightarrow q) \wedge \Diamond p) \rightarrow \Diamond q$ [inferred by *Chrysippus C11*]

Since $(p \Rightarrow q)$ iff $(\forall t)(\boldsymbol{T}(t, p) \rightarrow \boldsymbol{T}(t, q))$, we get $\Box(p\rightarrow q) \equiv \neg\Diamond(p\wedge\neg q) \equiv (p \Rightarrow q)$. Therefore, 1 (Prior's second premise) is equal to 4 (Øhrstrøm and Hasle's second premise).

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