Impossible Worlds and the Intensional Sense of ‘and’

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Abstract

In this paper I show that the ‘and’ in an argument like Lewis’ against concrete impossible worlds cannot be simply assumed to be extensional. An allegedly ‘and’-free argument against impossible worlds employing an alternative definition of ‘contradiction’ can be presented, but besides falling prey of the usual objections to the negation involved in it, such ‘and’-free argument is not quite so since it still needs some sort of premise-binding, thus intensional ‘and’ is needed and that suffices to block the argument at a stage prior to the steps about negation.

Keywords: Intensional ‘and’, contradiction, impossible worlds, ‘not’, Lewis

1. Introduction

Lewis’ argument against impossible worlds (Lewis 1986: 7) goes as follows. If worlds are concrete entities and the expression ‘at world *w*’ works as a restricting modifier—that is, if it restricts the quantifiers within its scope to parts of *w*—, then it should distribute through the extensional connectives.[1] Let us say that a modifier M distributes over an n-ary connective © if and only if, if $M(©(\phi_1,\ldots,\phi_n))$, then $©(M(\phi_1),\ldots,M(\phi_n))$. This means in particular that ‘At *w*, both *A* and not *A*’, where such a world *w* is

[1] Consider a sequence of sentences $\phi_1,\ldots,\phi_n$, another sentence ψ, and a valuation v that takes values in a collection V of values. An *n*-ary connective © is *extensional* if and only if the following condition is satisfied for any v:

$$\text{If } v(\phi_i) = v(\psi) \text{ then } v(©(\phi_i)) = v(©(\phi_1,\ldots,\phi_{i-1},\psi,\phi_{i+1},\ldots,\phi_n)).$$

© is *truth-functional* if and only if $v(©(\phi_i))$ is a function from V^n to V. Lewis speaks of truth-functional connectives, which is a rather strong requisite: truth-functionality implies extensionality (see Humberstone 2011: Ch. 3). The opposition truth-functional/intensional, very frequent in the literature, is not problematic in some cases. However, I will employ the proper opposition extensional/intensional.

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called then an *impossible world*, entails 'At *w*, A, and it is not the case that at *w*, A'. Thus, any contradiction at some world turns into an overt inconsistency at the actual world. But *pace* the dialetheist, there are no contradictions at the actual world, so there are no such impossible worlds.

This argument has been widely studied and virtually all its parts challenged, mostly trying to save impossible worlds for their theoretical applicability or even indispensability (see Nolan 1997: Sect. 2), or because they should stand along with possible worlds just by parity of reasoning (e.g. Vander Laan 1997, Beall and van Fraassen 2003). One option has been to reject Lewis' concretism. Thus, some think that worlds, whether possible or impossible, are abstract entities (e.g. Mares 1997, Vander Laan 1997). Others accept concretism for possible worlds but not for impossible worlds (e.g. Berto 2010, Divers 2002: Ch. 5). Others think that worlds different from the actual are not concrete but are not abstract either, at least in the usual sense of the term (e.g. Zalta 1997). Also, conceptions of logical (or, rather, modal) space radically different from Lewis' have been put forward (see Yagisawa 2010). All these are, however, realist views on worlds, so finally there are fully anti-realist views of impossible worlds (see Nolan 1997, Beall 2008).

As to other attempts to block Lewis' argument, some authors have maintained that Lewis begs the question against impossibilists in describing the nature of the modifier 'at *w*' (e.g. Lycan 1994). There is also the charge that using the law of non-contradiction in this context is question-begging (e.g. Yagisawa 1988) and, finally, those who would try to swallow the overt inconsistency at the actual world (e.g. Yagisawa 1988; Priest 2006a for several other reasons).

I argue here that the extensionality of 'and' in Lewis' original argument and almost all the subsequent discussion on it is an assumption that can be coherently challenged and rejected. Note that this is not a "desperate" move to block Lewis' argument: As mentioned above, there are already plenty of options in the market. I just want to emphasize that there is another option available, not necessarily in conflict with the others; that such option questions an early step of Lewis' argument, usually taken as safe; and that such option is worth pondering because of its implications for other logical, linguistic and metaphysical topics.

The plan of the paper is as follows. In Section 2 I show that reasons independent of the question about impossible worlds prevent from simply assuming that 'and' is extensional in Lewis' argument because 'and' is in general intensional. The next sections are devoted to answer some possible objections, in increasing degree of seriousness, against this appeal to the intensional sense of 'and' to block Lewis' argument. In Section 3 the charges of change of subject and that Lewis' argument holds at least for a sense of 'and' are dealt with. In Section 4 a supposedly 'and'-free argument against concrete impossible worlds is presented, which is still subject to already well-known objections to the extensionality of 'not', and I show that the reasons to support the intensional 'not' that blocks that argument belong to the family of reasons to support the intensional 'and' of Section 2. In Section 5 I argue that there is no reason to stick only to intensional 'not' although it suffices to block both arguments against concrete impossible worlds and, more importantly, that such 'and'-free argument is not quite so: Intensional 'and' is needed as a premise-binder and once properly analyzed the argument is blocked before the dubious steps concerning 'not'.

2. The Intensional Sense of 'and'

Lewis in his original argument against impossible worlds, and virtually all the authors discussing it, uncritically assumed the extensionality of 'and' so that the modifier 'at *w*' passes through it; or, more exactly, that 'and' satisfies the following property:

Distribution of M over 'and': For every A, B and restricting modifier M,

$$\frac{M(A \text{ and } B)}{M(A) \text{ and } M(B)}$$

Some inferences involving 'and', notoriously 'and'-elimination, have been recognized as problematic in this context, but almost always derivatively—see, e.g. Priest (2008: 172), Kiourti (2010: Ch. 4)—and, for reasons to be discussed in Sections 4 and 5, the extensionality of 'and' is ultimately left untouched.[2] Lewis (1982: 102ff) also considered failures of 'and'-introduction and 'and'-elimination for indexes that are nothing ontologically heavy but "(fragments of) corpuses of information". This comes as a "more conservative" approach to sentences that might be both true and false after his famous "dogmatic" rejection of sentences that are both true and false simpliciter. But in this context, such dogmatic rejection amounts to say that the indexes in which sentences are both true and false cannot be concrete worlds, but that is precisely what is at stake.

However, a sense of 'and' that does not satisfy *Distribution of M over 'and'* has been introduced and defended on grounds other than contradictions and impossible worlds: Fusion, also called sometimes 'intensional', 'group-theoretical', or 'multiplicative' conjunction in the tradition of substructural logics, which serves to express the idea that the joint content of the conjuncts is needed to entail something. Humberstone (2011, esp. Ch. 5) has criticized the introduction of new connectives, especially for different senses of 'and', on the basis that most of the times matters pragmatic are confused with matters semantic and no real logical need for a new 'and' is put forward. But the picture for fusion seems promising. If an indicative conditional is required to connect contents (semantic, informational or whatnot) beyond those expressible by truth values alone, as would happen if it embodies in an object language a suitable notion of entailment, then a conditional like 'If I like sandwiches then this is an amazing journal.' might not be true even if both antecedent and consequent are true. But Read (1981; 2003) has argued that if an indicative conditional connecting more than truth values is true, then the 'and' of an inference like

$$\frac{\text{Both } A \text{ and } B}{\text{So if } A \text{ then } B}$$

[2] With the notable exceptions of Nolan 1997 and Yagisawa 2010, to be discussed at the end of this section.

cannot be an extensional connective, for the premise could be true and the conclusion false.

Read (1981) has also claimed that phrasings like ‘It cannot be both *A* and not *B*’ do not express an impossibility but rather the inferential connection between ‘*A*’ and ‘*B*’. In that case, ‘It cannot be both *A* and not *B*’ would imply ‘If *A* then *B*’, so ‘It cannot be both that the premises of an argument are true and the conclusion false’ would imply ‘If the premises of an argument are true then the conclusion is true’. In general, this ‘and’ in the very characterization of validity should be non-extensional, as would be the ‘and’ and other premise-binding devices in an argument.[3]

Such interaction between indicative conditionals and ‘and’ has appeared independently in the debate about impossible worlds. Let us suppose that an impossible world is more generally a world where logic is different from the actual world, not only one where sentences of the form ‘Both *A* and not *A*’ are true. In particular, sentences of the form ‘If *A* then *A*’ may fail to be true. Such a conditional still comes with a residual, $\circ$ (the double bar indicates that the inference goes both ways):

$$\frac{A \text{ implies that if } B \text{ then } C}{A \circ B \text{ implies } C}$$

i.e. a sort of conjunction, just read ‘and’ for $\circ$, which nonetheless should be in general not extensional to keep company the failure of ‘If *A* then *A*’.[4]

Fusion is a very general proposition-binder, and especially a premise-binder, that can work as the formal counterpart of that ‘and’.[5] Every proposition *A* is evaluated at an index *i*, which is a (possibly empty) *multiset* of conditions of evaluation additional to its truth value. A *binding*, *fusion* or *intensional conjunction* of the propositions *A* and *B*, denoted here ‘$A \circ B$’, at an index *i* has the same content as the multi-union in *i* of the content of *A* at some index *j* and the content of *B* at some index *k,* both of the latter related to *i*.[6] More perspicuously,

[3] The intensional ‘and’ has also served to express relevance as a necessary component of validity understood as truth-preservation rather than a condition to be added to it; see Read 2003. Moreover, appealing to it can be helpful to deal with paradoxes like Curry or the so-called ‘paradoxes of validity’, as in Priest 2015.

[4] For more details, see Slaney 1990. That this conditional comes with such a residual goes against Priest’s (2008: 172) idea that if an impossible world is a world where the laws of logic are different (from those of a certain designated world, presumably the actual one, where presumably classical logic holds), then the behavior of conditionals, and only of them, change at impossible worlds, because conditionals express the laws of logic: If conditionals are affected, conjunctions are affected as well, and so it is not true that “[t]here are no [non-normal, impossible] worlds at which $A \wedge B$ is true, but *A* is not”.

[5] In what follows I stick closely to Mares’ simple and intuitive explanation of fusion (cf. Mares 2012).

[6] A multiset is like a set, except that it may contain identical elements repeated a finite number of times. If *X* and *Y* are multisets then, for every element *a*, if it appears *n* times in *X* and *m* times in *Y*, then it appears $n+m$ times in the multi-union of *X* and *Y*.

$$\frac{A \text{ and } B}{A, B}$$

is not valid, because sentences in general not only have a truth value, but additional content provided by the indexes in which they are uttered or evaluated. The more general form of the inference above helps to show why it is not valid:

$$\frac{(A_j \text{ and } B_k)_i}{A_{ji}, B_{ki}}$$

Thus, from $(A_j \text{ and } B_k)_i$ one cannot infer A_i (or B_i), but at most A_{ji} (or B_{ki}).[7]

In classical zero-order logic, for all practical purposes there is but only one index, the empty index, so $i = j = k$ and the contents bound in that index are present separately in that index and that validates the usual rule of conjunction-elimination. Start with the general version

$$\frac{(A_j \text{ and } B_k)_i}{A_{ji}, B_{ki}}$$

Since $i = j = k$,

$$\frac{(A_i \text{ and } B_i)_i}{Aii, Bii}$$

hence

$$\frac{(A \text{ and } B)}{A, B}$$

because the multi-union of empty multisets is empty, $ii = i$, all indexes are identical and hence can be obviated.

Nonetheless, the more general version

$$\frac{\text{At } i, A \text{ and } B}{\text{At } i\ A \text{ and at } i, B}$$

is not valid, because what there is at i is the multi-union of contents that may be present only at indexes different from *i.* When the indexes are worlds, even concrete worlds like Lewis', *Distribution of M over 'and'* needs not hold.

[7] Here is a rather simple example. Suppose that A holds at j, which is the multiset of conditions of evaluation [b, c, c, d], and that B holds at k, which is the multiset of conditions of evaluation [a, a, c]. Then 'A and B' holds at the index i = [a, a, b, c, c, c, d], but it is not true that A (or B) alone holds at [a, a, b, c, c, c, d]. Indeed, one of the conditions in [a, a, c] (respectively, [b, c, c, d]) might prevent having A (respectively, B) alone, as in the case of connexive logic mentioned below.

Thus, given that indicative conditionals in general are not extensional—or, at the very least, in general they are not extensional in impossible worlds—, 'and' in general is not extensional, either. But if 'and' in general is not extensional, *Distribution of M over 'and'* needs not hold. Also, if 'and' in general is not extensional, there is no reason to suppose that it is in a sentence of the form 'A and not A' which is true (by hypothesis) at an impossible world. And if there is no reason to suppose that 'and' is extensional in an impossible world, then 'at w' does not necessarily distribute through it. But if it does not, Lewis' original argument against (concrete) impossible worlds does not run.

Someone might retort that, even if all this formal account connecting conjunction and the conditional is sound, no actual example of the failure of 'and'-elimination has been provided. Of course, that 'and'-elimination is invalid does not mean that every instance of it should be rejected. No doubt 'There is a laptop on the desk' validly follows from "There are a laptop and a tablet on the desk', but if even a single instance of 'and'-elimination can be found in which the conclusion fails to follow from the premises, then that will be sufficient to show that 'and'-elimination is not valid.

In fact, there are at least three classes of such instances. The first is provided by the cancellation theory of negation (see Priest 2006: 31ff for an overview). According to it, an assertion says one thing, and its denial withdraws it. Thus, a contradiction cancels its content out and leaves nothing behind. Therefore, nothing follows from a contradiction, in particular, none of its conjuncts. The second class is provided by inferences of the following form, familiar from connexive logic (cf. Thompson 1991):

$$\frac{A\text{, and } A \text{ does not follow from } A}{A}$$

But this means that A follows from *A* even under the condition that it does not. Thus, the inference from '*A* and *B*' to '*A*' is rejected on the grounds that *'B'* might assert something that would countermand the inference from *'A'* to *'A'*. Finally, consider this counterexample introduced by Gillian Russell (in preparation). Let us say that a literal, i.e. an atomic proposition or its negation, is solo if and only if it is not embedded in a conjunction. Then the following inference seems to go from true premises to a false conclusion

$$\frac{\text{This very sentence is not SOLO and snow is white}}{\text{This very sentence is not SOLO.}}$$

In all fairness, at least Nolan (1997) and Yagisawa (2010) have regarded *Distribution of M over 'and'*, as well as other classical inferences connecting 'and' and conditionals discussed above, problematic when it comes to impossible worlds. The distinctiveness of my approach is twofold. First, I relate the discussion explicitly to Lewis' argument against impossible worlds, whereas those principles are just samples in Nolan's broader discussion of what should go in an 'ultralogic' to reason about any kind of situation, including impossible worlds. Second, Nolan thinks that the inferences might fail to hold only at very strange situations, whereas I regard the counterexamples to those inferences rather natural, at least as natural as are those for the intensional conditional. Yagisawa (2010: 183f) makes the failure of *Distribution of M over 'and'* a feature only of impossible worlds, almost as if 'and' could have arbitrary truth-conditions at an impossible world, but although the idea that impossible worlds are semantically arbitrary at least for some connectives is very widespread—see for example the reports in Beall (2009)—, such arbitrariness is not necessary to make sense of the failure of certain classical inferences concerning 'and', what is needed is just a bit of generalization.[8]

3. Changing the Subject and Two Kinds of 'and' for Impossible Worlds

It can be objected that the approach above is merely an *ignoratio elenchi*, that Lewis' was discussing extensional 'and' and that his argument indeed works for it. I think this is wrong even as an exegetical point. Lewis argues that if sentences of the form 'A and not A' are true at some worlds, then they are true at the actual world, which would be wrong and then the hypothesis should be rejected. In doing that he simply assumed that connectives are extensional, that is, Lewis does not argue that there are no concrete impossible worlds for some specific formalizations of sentences of the form '*A* and not *A*', but that there are no concrete impossible worlds, period. I think this is the usual underlying dialectical approach to Lewis' argument. For example, when Lycan (1994) or Kiourti (2010) challenge the idea that 'at w' passes through 'not', they do not think of themselves nor are taken by their interlocutors as changing the subject of Lewis argument, but as providing a better understanding of the logic of the sentences of the form '*A* and not *A*' and its consequences for the status of impossible worlds. The same goes for the analysis of 'and'.

The second objection strengthens the previous one and considers the case of a language suitable to talk about impossible worlds with both kinds of 'and': Even if

[8] The appeal to an intensional 'and' has consequences for other discussions in the vicinity. In a broader discussion of the appropriate semantic clauses for possibility and impossibility once impossible worlds are admitted, Divers (2002: Ch. 5) says that the usual clause for impossibilities

(IP) *A* is impossible if and only if there is no world, *w*, such that at *w*, *A*

would be false, for impossibilities hold at impossible worlds, and that amending (IP) in the following way

(IP*) *A* is impossible if and only if there is an impossible world, *w*, such that at *w*, *A*

would be useless: Suppose $A \wedge \neg A$ holds at an impossible world *w*. Then presumably each conjunct holds at *w*, that is, at *w*, *A*, and at *w*, $\neg A$. But since each conjunct holds at an impossible world, each of them is impossible, by (IP*), although clearly in general they are not. Again, it is presupposed that the conjunction is extensional; if it is not, this argument against (IP*) does not work. I am not saying that (IP*) is the right semantic clause for impossibility; what the proper stance about the semantic clauses for possibility and impossibility should be is a more general problem that I will not discuss here.

Lewis' argument does not work for contradictions with an intensional 'and', it still works for contradictions in which 'and' is extensional, so having both kinds of 'and' at an impossible world would turn at least some contradictions there into contradictions at the actual world. Two interrelated answers can be given to this objection. The first one is that in the case described, Lewis' argument would only show that there are no concrete impossible worlds with both kinds of 'and', not that there are no concrete impossible worlds at all. An argument against all concrete impossible worlds would require an argument for the necessary presence of both kinds of 'and' in the language corresponding to an impossible world. In the absence of such argument for necessarily having both kinds of 'and', that Lewisian argument would be precisely the argument for the inadmissibility of the extensional 'and' in a language for impossible worlds.

The second reply is that an argument for the inadmissibility of the extensional 'and' in languages for impossible worlds could be an excessively strong demand. What would be needed is at most an argument for the inappropriateness of the extensional 'and' to represent sentences of the form '*A* and not *A*'. This is an idea not uncommon among thinkers of contradictions. In several places of *Metaphysics* Aristotle attributed to the Heracliteans the thesis that 'All contradictions (and only them) are true' and reconstructed their views as follows.[9] Everything is in state of flux at every moment, so and a thing cannot be described truly to be an *F* because it would be to fix it, and the same considerations are made for not being an F, but it can be described truly and fully as being an *F* as well as not being an *F*. Hence all contradictions, but none of their components separately, are true, because it would be the only way to capture the changing nature of things (cf. *Metaphysics* 1005^{b}25, 1007^{b}26, 1012^{a}25). More recently, when discussing how contradictions could be observed, Paul Kabay (2010: 110) proposes that contradictions are different from other conjunctions, that they are more like "single whole entities", not a "unified structure with distinct parts", and then he claims that contradictions, if true, would involve the same thing in all the same respects, as Aristotle stated, so there would be nothing to separate. Actually, Priest (2006: 11) also notes that Aristotle in Prior Analytics 57b3 cannot accept 'and'-elimination for contradictions since he claims that "contradictories cannot both entail the same thing". Finally, that the conjunction of contradictions involved in Hegel's dialectic and theses like that of the unity and identity of opposites is different from ordinary conjunctions was raised by some commentators (e.g. Wetter 1958; Havas 1981; Priest 1989: 397).

A final objection would say again that, in spite of what the thinkers above could argue, there is a change of subject, because sentences of the form 'A and not A', where 'and' is intensional, are not real contradictions; real contradictions are sentences of the form 'A and not A' where 'and' is extensional, and Lewis' is arguing against those. I have pointed out in the preceding section that this is exegetically

[9] Priest sometimes conflates trivialism—'Everything is true'—with a version of Heracliteanism—'All contradictions are true'—(cf. Priest 2007: 131), although sometimes he acknowledges that the identification depends on certain assumptions, notoriously 'and'-elimination (see Priest 2006: 56). Aristotle too thought that semantic Heracliteanism could be equated with trivialism, but he was more cautious; cf. *Metaphysics* 1012^{a}25.

incorrect. But, moreover, I have showed above that extensional 'and' can be seen as a limit, extremely idealized case of the general case but where there is but one, dispensable index. Further argumentation is needed to claim that only conjunctions (and negations) uttered or evaluated at a certain single index are the "real" conjunctions and negations, and so the only connectives that can produce "real" contradictions.

Thus, provided the correctness of the independent arguments for the idea that 'and' is in general intensional and the views of thinkers of contradictions just mentioned, what needs to be argued for is the idea that sentences of the form '*A* and not *A*' are correctly formalized using an extensional 'and'—not to mention the stronger claim that only it provides the correct formalization of those sentences—even if it is available in the language.

4. Another Definition of 'Contradiction' and the Intensionality of 'not'

Even if Lewis uses the definition of a contradiction as a sentence of the form 'A and not A' and it is the way it has usually been discussed, an apparently 'and'-free argument can be formulated taking the definition as a pair of sentences '*A*, not *A*'.[10] In that case, the appeal to the intensional 'and' would seem useless, for the rule

For every *A*, *B* and restricting modifier *M*,

Distribution of M over 'and': $$\frac{M(A, B)}{M(A),\ M(B)}$$

applied to contradictions as pairs

$$\frac{M(A, \text{not } A)}{M(A),\ M(\text{not } A)}$$

sounds good: If we have a pair of contradictories at M, it sounds plausible that we have at M each of the members of the pair, unlike the case of contradictions of the form '*A* and not *A*'.

However, this move leaves Lewis' argument at the mercy of already known objections to others of his assumptions. If the hypothesis of this kind of impossible worlds is taken seriously, there is no reason to grant that 'at w' passes through 'not'

[10] Stalnaker's version of the argument against impossible worlds uses the latter option (cf. Stalnaker 1996/2002). Lewis (1986) does not consider contradictions of the form '*A*, not *A*'. This is not accidental. His discussion of *ex falso quodlibet* in Lewis (1982: 104) makes clear that he considered that the pair version may be susceptible to counterexamples because the separate premises *A*, not *A* might track contents from different indexes, while he was confident that the single premise '*A* and not *A*' could avoid that. However, as discussed in Section 2, the intensional 'and' also tracks contents from different indexes.

to produce the contradiction at the actual world, i.e. that 'not' satisfies the following property (see Kiourti 2010: Ch. 4):

For every A and restricting modifier M,

Distribution[11] *of M over 'not'*:

$$\frac{M(\text{not } A)}{\text{Not } (M\,A)}$$

This feature of 'not' is as related to conditionals as are the motivations for the intensional 'and'. If an indicative conditional is required to connect contents beyond those expressible by truth values alone, some indicative conditionals of the forms 'If A and not A then B' and 'If A then B or not B' are not true. That might be achieved by an intensional negation of A, denoted here '$\ominus A$'.[12] $\ominus A$ is true at an index of evaluation i if and only if A is false at some index j maximal with respect to all the indexes compatible with i. More perspicuously, in the more general version

$$\frac{\text{At } i, \text{ not } A}{\text{Not at } i, A}$$

is not valid, because the truth of 'Not A' at i is the untruth of A at an index k that may be different from i. (I am sure the reader can give the explicit version of this as was done for 'and'.) When the indexes are worlds, even concrete worlds like Lewis', *Distribution of M over 'not'* needs not hold. Remember that in classical logic, for all practical purposes there is but only one index, so $\ominus A$ is true if and only if A is untrue at that single index.

5. Intensional 'and' Strikes Back

The possibility of using the intensional 'not' to block Lewis' argument raises a further objection to the approach advocating the intensional sense of 'and'. Since 'not' appears in both arguments, is not it "the only relevant" logical notion, as Stalnaker's (1996/2002: 58) Louis would say? And, moreover, if intensional 'not' suffices to block the arguments against concrete impossible worlds with either definition of contradiction, should not the changes be kept at minimum and avoid introducing

[11] Some people might feel that this is not really a case of distribution over a connective, but rather a sort of commutativity or other phenomenon. However, unary connectives like negation can be present in special cases of distribution as it was defined right in Section 1.

[12] I will follow Restall's explanation of this negation (cf. Restall 1999). The required notions are as follows: Suppose that there are accessibility relations between indexes, one of which constitutes a partial order and that can be denoted '$\leq$'. That an index i is *maximal* with respect to an index j means that for every index x, if $j \leq x$ then $x \leq i$. What the compatibility of j with i exactly means depends on substantial philosophical ideas, but at least the following characterization might be admitted without much problems: j is *compatible* with i if (1) Rij, (2) there is another relation between i and j, denoted 'Cij', such that C is symmetric, non-reflexive and that, for all indexes x and y, if Cij, $x \leq i$ and $y \leq j$, then Cxy.

the intensional 'and' which would serve at most for one version? I can think of at least two answers to these questions. First, if the arguments for intensional 'and' from the nature of indicative conditionals reconstructed in Section 2 are right and if defining a contradiction as a sentence of the form '*A* and not *A*' is on equal footing with defining it as a certain kind of pair, intensional 'and' can be legitimately invoked when in presence of contradictions of the mentioned form, as in Lewis' original argument and most of the subsequent discussion.[13]

But the most important reason for not sticking only to the intensional 'not' is that the claims that intensional 'and' is irrelevant for *Distribution of M over ','*, or that 'not' is the only relevant logical notion in the 'and'-free argument, are contentious, for the argument depends on the way of interpreting bound premises. Again, if indicative conditionals connect contents beyond those expressible by truth values alone, and if in one's logic logical consequence and the conditional are related by a "deduction theorem", that is, B is a logical consequence from $A_1,\dots, A_n$ if and only if the corresponding conditional 'If A_1 and… and A_n then B' is a theorem, those 'and's must be in general intensional to keep company the indicative conditional; see Read (1988: Ch. 3); Slaney (1990). But that would mean that also the commas in '$A_1,\dots, A_n$' are intensional premise-binders.[14] Thus, even if the argument against concrete impossible worlds started 'At w, A, not A', there are reasons to think that a modifier like 'at w' does not pass through comma or any other premise-binder. The intensional premise-binding has thus an occurrence in the argument prior to that of the intensional 'not' and has to be analyzed first. But, once the premise-binder is properly analyzed, Lewis argument can be resisted at an earlier stage.

6. Conclusion

Intensional 'and' represents another option in the market to block Lewis' argument against impossible worlds, whether under the assumption that contradictions are sentences of the form '*A* and not *A*' or under the alternative definition of contradictions as certain pairs, as in Stalnaker's version of the argument. It not only does not compete with the well-known strategy of being more precise about 'not' in the context of impossible worlds, but together with this it grist to the mill of those who

[13] Kiourti (2010: Ch. 4), building upon some concerns expressed by Nolan (1997), considers two possible problems that might constrain the number of admissible intensional notions. The first one is that the intensional truth-conditions of connectives "no longer [would] allow us to break these down to their individual components at such worlds, treating them instead as atomic predicates of worlds." This "might seem extreme", as she says, but her own answer, which seems right, is that it seems less extreme if one notes that this is a feature of impossible worlds that not necessarily spills over into any other worlds. The second worry, which she considers more pressing and leads her to adopt just the intensional 'not', is that one might fall short of principles to reason about an entire ontology comprising both possible and impossible worlds. But that is not the case; intensional connectives are disassociated from certain inferences but not from all of them.

[14] Like fusion, this feature of the comma has been widely studied in the so-called substructural logics; see for example Restall 2000, Ch. 2.

think that it is possible to have an extended modal realism *á la* Yagisawa without contradictions at the actual world, by blocking Lewis' argument at an earlier step.
The remarks about 'and' would also serve to block one of the most ancient arguments against true contradictions, the argument for explosion, popularized by another Lewis. The most common strategies are either rejecting Disjunctive Syllogism or appealing to an intensional sense of 'or' (see Read 1988: Ch. 2). But if there is no reason to suppose that an 'and', especially that of a contradiction, is extensional, or if premise-binding is tighter than usually thought, the argument can be blocked already at the step following the assumption(s).

The appearance of intensional 'and' in the context of impossible worlds invites one to think about further implications of this notion for the philosophy of logic, especially when it comes to the understanding of some connectives and on a possible debate about the preferability of one definition of 'contradiction' over the other.[15]

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