S4 to 5D

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Abstract

The modal logical axiom 4 is widely accepted. It is the characteristic axiom of the modal logical system S4, which is subsumed under the most popular modal logical system S5. Axiom 4 is equivalent to $\Diamond\Diamond P \rightarrow \Diamond P$ ("If possibly possibly *P*, then possibly *P*"), which requires that the accessibility relation between worlds be transitive.

There is a powerful argument (Hugh Chandler 1976, Nathan Salmon 1981, 1989) against axiom 4. It rests on the thought that an ordinary object could have had a slightly different origin from its actual origin but could not have had an origin very different from its actual origin. By constructing a sorites-like sequence of possible worlds at which the origin of a given object shifts incrementally along the sequence, the argument concludes that accessibility is not transitive, i.e. that what is possibly possible may not be possible.

A recent attempt to defend S4 from this argument (Murray and Wilson 2012) proposes that we abandon the absolute notion of possibility and instead accept a world-indexed notion of possibility; each world comes with its own version of possibility.

I offer a different defense of S4, which preserves both axiom 4 and the absoluteness of possibility. Its key move is to postulate objects as extended not only in physical space-time but in logical space as well, that is, as "five-dimensional" worms. Since S4 and the absolute notion of possibility are very intuitive, quite useful, and widely well regarded, and since my proposal saves both of them, I take the proposal to constitute an argument in favor of "five-dimensionalism."

Keywords: Modality, S4, Origin Essentialism, Five-Dimensionalism, Impossible World.

1. Introduction

S5 is the most popular modal logical system among modal metaphysicians. S4 is weaker than S5. So, anyone who accepts S5 should also accept S4. But there is trouble with S4. Or so argue Hugh S. Chandler and Nathan Salmon. Chandler's argument is directed against Alvin Plantinga's claim that nothing is possible at some possible worlds and not possible at others. Chandler aims to establish "that what is possible

varies from world to world."[1] Salmon takes Chandler's argument and elaborates on it more broadly as an argument against S4.[2] Even though it is Salmon, not Chandler, who explicitly targets S4, I shall be concerned with Chandler's original version of the argument for its simplicity. The general thrust of my discussion applies equally well to Salmon's version.

Chandler's argument threatens the characteristic axiom of S4, namely:

Axiom 4: $\diamond\diamond P \rightarrow \diamond P$.

It says that whatever is possibly possible is possible.[3] Independently of commitment to stronger S5, this axiom seems well worth saving by itself. Here are two examples illustrating its plausibility:

(i)   I have no child but could have had one. If I had a child, that child could have had a child. So, I could have had a grandchild.
(ii) There is a physical particle which does not split but could have split into two particles. If it had so split, each of the two resulting particles could also have split into two particles, producing four further particles in total. So, there could have been four particles instead of just one.

These are just examples and do not amount to an argument in favor of Axiom 4, but their overwhelming natural plausibility should strongly encourage us to attempt search for a way to save Axiom 4 from any objection against it. That is the spirit in which I approach Chandler's argument.

That spirit is shared by Adam Murray and Jessica Wilson, who propose a way to save Axiom 4.[4] Their rescue attempt, however, comes at a serious cost and also seems ineffective. I wish to propose a different way to save Axiom 4 without the cost and with effectiveness.

2. Preliminaries

According to standard modal logic, truth is indexed to a world, and truth of a possibility at a world is truth at an accessible world:

(PT): $\diamond P$ is true at a world *w* if and only if P is true at some world accessible from *w*.

In general accessibility is any dyadic relation between worlds, but given (PT), Axiom 4 constrains it to be transitive: for any worlds *w* and *w′*, if *w′* is accessible from some world that is accessible from *w*, then *w′* is accessible from *w*. Intuitively, accessibility is intended to be relative possibility: *w′* is accessible from *w* if and only if all that holds at *w′* is possible relative to *w*. This is strictly just an intuitive idea, for if it were taken seriously as a definition of accessibility, possibility would be defined in terms of relative possibility, and the latter would remain in need of further definition if we wanted

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[1] Chandler 1976: 106.
[2] Salmon 1989.
[3] An equivalent formulation of Axiom 4 is: P →   P (whatever is necessary is necessarily necessary).
[4] Murray and Wilson 2012.

to complete definitions of all modal notions. (I am assuming that relative possibility is a modal notion.) This will become important when we evaluate the proposal by Murray and Wilson.

The possibility operator $\diamondsuit$ may be interpreted in different ways: logical possibility, metaphysical possibility, physical possibility, human psychological possibility, legal possibility (for a given society), etc. Chandler focuses on metaphysical possibility, the subject matter of Plantinga's claim.[5] Salmon's elaboration on Chandler's argument and the criticism by Murray and Wilson do not deviate from this focus. My discussion will also be strictly about metaphysical possibility, and no other kind of possibility will be considered in this paper.

3. Argument Against Axiom 4

A bicycle is an artifact which is manufactured out of, or originates from, many parts. If a particular bicycle in fact originated from particular parts, then that very same bicycle could possibly have originated from the same particular parts except for one spoke; in place of that spoke, a completely different spoke might have been used to manufacture a bicycle numerically identical with the original.

Some might wish to deny this and insist that even if all other original parts were used in the same way as for the original bicycle, if one spoke were different, then the resulting bicycle would not be numerically identical with the original bicycle. If such obstinate essentialism concerning origin is accepted, Chandler's argument is blocked at the outset.[6] It is not my intention to block Chandler's argument this way; neither is it the intention of Murray and Wilson. If a spoke would make a difference to the numerical identity of the resulting bicycle, there seems to be no principled reason to deny that a small part of a spoke would also make a difference. But if so, it seems that a large molecule would make a difference, too. But if a molecule would, why not an atom? And it seems implausible to insist that one atomic difference in origin would destroy the numerical identity of the manufactured bicycle.

On the other hand, if no original parts had been used to manufacture a bicycle except for one original spoke, then the resulting bicycle would not have been numerically identical with the original bicycle. It is unclear how many of the original parts should have been used to retain the numerical identity of the original bicycle. To avoid deciding this tangential issue with a bicycle, or any other familiar kind of object, and simplify discussion, Chandler conjures up an imaginary kind of object, which he calls *alpha*. He stipulates that any object of this kind—any alpha—originates from three parts and that it is possible for any alpha that in fact originated from three particular parts to have originated from two of those parts plus a different third part, but not from one of those parts plus two different parts, or from none of those parts. Thus, alphas are compound material objects with a very unusual condition for origin: for any alpha x, if x originated from matter m, then x could not have originated from matter two-thirds or more different from m. No familiar compound material object

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[5] Or possibility "in a broad logical sense"; see Chandler 1976: 106.
[6] The phrase "obstinate essentialism" is due to Salmon.

has such a simple and sharp condition for origin. This unfamiliar nature of alphas should not discourage us from going along with Chandler's scenario. Far from it, we should welcome the simplification Chandler brings forth by the introduction of alphas, as a measure that helps us focus our attention squarely on the core issue of the transitivity of accessibility without distraction.[7]

With this simplifying assumption, Chandler considers a particular alpha, which he calls *Alfred*. Let us say that at a world w_0 Alfred exists and originated from matter consisting of three particular parts, 1-2-3. So at w_0 Alfred could have originated from 4-2-3, where 4 is distinct from 1.[8] That is, at some world w_1 accessible from w_0, Alfred exists and originated from 4-2-3. Since at w_1 Alfred originated from these three parts, at w_1 Alfred could have originated from two of them plus a new part, say, from 4-5-3, where 5 is distinct from 2 and from 1. That is, at some world w_2 accessible from w_1, Alfred exists and originated from 4-5-3.

Suppose for *reductio* that w_2 is accessible from w_0. Then Alfred exists and originated from 4-5-3 at a world accessible from w_0, which means that at w_0 Alfred could have originated from 4-5-3. But being an alpha and having originated from 1-2-3 at w_0, Alfred at w_0 could not have originated from 4-5-3. A contradiction! Therefore, w_2 is inaccessible from w_0. Since w_2 is accessible from w_1, which is accessible from w_0, accessibility is not transitive. This is Chandler's argument.

Let us put Chandler's argument in a regimented way to reveal its logical structure:

P: Alfred originated from 4-5-3.
1. P is not true at any world accessible from w_0.
2. P is not possibly true at w_0.
3. P is true at w_2, and w_2 is accessible from w_1, which is accessible from w_0.
4. P is possibly possibly true at w_0.
5. Some proposition, viz., P, is not possibly true but possibly possibly true, at w_0.
6. Accessibility is not transitive, i.e., Axiom 4 is false.

1 and 3 are true, 2 follows from 1, 4 follows from 3, 5 follows from 2 and 4, and 6 follows from 5. Or so claims Chandler.

It is important to note that the alpha which exists at w_2 and originated from 4-5-3 is supposed to be indeed Alfred, and not some other alpha. If it were some alpha other than Alfred, then that alpha's having originated from 4-5-3 would not make w_2 inaccessible from w_0. It is perfectly possible at w_0 that some alpha other than Alfred exists and originated from 4-5-3.

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[7] To see clearly that vagueness in the condition of origin is largely a distraction, observe that Chandler's argument can be easily adapted according to a scenario incorporating vagueness as long as the vagueness permits the starting point and the end point in the *sorites*-like series of minutely shifting origin of a given object, to be an uncontroversially possible case of origination and an uncontroversially impossible case of origination, respectively.

[8] It goes without saying that 4 is also distinct from 2 and 3. Similar suppositions will not be noted explicitly henceforth.

It is also important to bear clearly in mind that only one kind of possibility, viz., metaphysical possibility, is in question and that therefore only one accessibility relation figures in the scenario used in the argument. It is intended that the accessibility relation holding between w_1 and w_0 is the very same accessibility relation holding between w_2 and w_1, and this very same accessibility relation is argued to fail between w_2 and w_0. If this were not so, the argument would not succeed in exhibiting a single non-transitive accessibility relation.

4. Attempt to Save Axiom 4

Murray and Wilson do not consider Chandler's Alfred example but discuss Salmon's different example instead. Their response to Salmon's version of Chandler-ish argument, however, can be adapted straightforwardly to become a response to Chandler's original argument.

Murray and Wilson in effect object to Premise 3 in Chandler's argument above. Their position is that 3 is ambiguous, for "accessible" is used ambiguously. And this ambiguity is inherited by 4 and 5 so that 6 does not follow. They deny that one single accessibility relation figures in the scenario and deny that one single notion of possibility is iterated in 4 or 5; that is how they propose to block the argument.

They phrase this move in terms of their own technical notion, *considered as indicatively actual*. When w_0 is considered as indicatively actual, Alfred could have originated from 4-2-3, but not from 4-5-3, whereas when w_1 is considered as indicatively actual, Alfred could have originated from 4-5-3. Murray and Wilson consider themselves as borrowing this technical notion from two-dimensional semantics and using it not so much for semantic or epistemic purposes as for metaphysical purposes. They propose that metaphysical possibility is relative to a world considered as indicatively actual. If I understand their proposal correctly, this means, as I have already indicated, that they wish to block the argument by denying that one single accessibility relation is involved throughout the scenario. We may put it this way: When we adopt the point of view of w_0 at the beginning and say that Alfred could have originated from such-and-such parts, we are resorting to one accessibility relation, which we might call *accessibility$_0$*, and say that w_1 is accessible$_0$ from w_0. When we move on to adopt the point of view of w_1, however, we shift to a different accessibility relation, *accessibility$_1$*, and say that w_2 is accessible$_1$ from w_1. And when we say that Alfred at w_2 is impossible, we are returning to the point of view of w_0 and saying that w_2 is inaccessible$_0$ from w_0. There is no absolute accessibility common to the point of view of an arbitrary world, but there are only world-relative accessibility relations, one for each world (considered as indicatively actual). To each such accessibility relation corresponds a distinct notion of possibility: possibility$_0$ to accessibility$_0$, and possibility$_1$ to accessibility$_1$.

Here is how Murray and Wilson would see Chandler's argument:

P: Alfred originated from 4-5-3.
1'. P is not true at any world accessible$_0$ from w_0.
2'. P is not possibly$_0$ true at w_0.
3'. P is true at w_2, and w_2 is accessible$_1$ from w_1, which is accessible$_0$ from w_0.

4'. P is possibly$_0$ possibly$_1$ true at w_0.

5'. Some proposition, viz., P, is not possibly$_0$ true but possibly$_0$ possibly$_1$ true, at w_0.

6'. Accessibility is not transitive, i.e., Axiom 4 is false.

Clearly, 6' does not follow from 5'. Moreover, according to Murray and Wilson, it is not that P is possibly$_0$ possibly$_0$ true at w_0, it is not that P is possibly$_1$ possibly$_1$ true at w_0, and it is not that P is possibly$_k$ possibly$_k$ true at w_0, for any k; so there is no counterexample to Axiom 4 in Chandler's scenario.

Murray and Wilson's move to preserve Axiom 4 in the face of Chandler's argument is a radical move. It proliferates accessibility relations for metaphysical possibility, hence it proliferates varieties of possibility all of which fall under the umbrella notion of metaphysical possibility. Indeed, it produces as many varieties of metaphysical possibility as there are worlds to be considered as indicatively actual. But this seems undesirable. Metaphysical possibility is a kind of possibility among many different kinds of possibility, but it seems that there is only one kind of possibility that is metaphysical possibility, and it certainly seems that there are not as many varieties of metaphysical possibility as there are worlds (eligible to be considered as indicatively actual).

As we saw in the opening paragraph, Chandler claims against Plantinga that what is metaphysically possible "varies from world to world." Murray and Wilson are presumably on Plantinga's side but they claim that metaphysical possibility itself varies from world to world; once a particular variety of metaphysical possibility is fixed, what is possible does not vary from world to world, but what variety of metaphysical possibility is in question to begin with does so vary. This hardly seems like much of a defense of Plantinga, or Axiom 4.

Also, even if there are many different varieties of metaphysical possibility and corresponding accessibility relations, Chandler's argument seems to go through unscathed after all. Suppose with Murray and Wilson that metaphysical possibility is always relative to a world (considered as indicatively actual). Take the metaphysical possibility relative to the actual world w_0 (by considering w_0 as indicatively actual). This determines a particular accessibility relation, R. Everyone agrees that w_1 is R to w_0. How about w_2 and w_1? Is w_2 R to w_1? Alfred originated from 4-2-3 at w_1 and originated from 4-5-3 at w_2. Since Alfred is an alpha at w_0, R is such that a world at which Alfred originated from matter that is two-thirds identical with the matter from which Alfred originated at a given world is R to that given world. So w_2 is R to w_1.

Do not be fooled into thinking that w_2 is not R to w_1 on the ground that at w_2 Alfred originated from the matter only one-third identical with the matter from which Alfred originated at w_0. Considering w_0 as indicatively actual only determines the variety of metaphysical possibility, i.e., only fixes the accessibility relation to be R. It does not make the original matter of Alfred at w_0 be the object of comparison to the original matter of Alfred at w_2 when considering the question whether w_2 is R to w_1.

Since the question is whether R holds between w_2 and w_1, not whether R holds between w_2 and w_0, the object of comparison should be the original matter of Alfred at w_1.[9]

At the same time, since Alfred's original matter at w_2 is only one-third identical with Alfred's original matter at w_0, w_2 is not R to w_0. Thus, w_1 is R to w_0, w_2 is R to w_1, but w_2 is not R to w_0. Therefore, R is not transitive.

Although I am not in favor of Murray and Wilson's application of two-dimensional semantics according to which each world gives rise to a different variety of metaphysical possibility, there seems to be something attractive about their radical approach. We can use what I take to be the underlying spirit in which they offer the core idea of *considering a world as indicatively actual*. As we move from w_0 to w_1, or from w_1 to w_2, when considering how Alfred could or could not have originated, something metaphysical shifts. What shifts is not the variety of possibility, but reference. Or so I claim. In my view, a proper response to Chandler's argument will not only preserve Axiom 4 but also give us an opportunity to learn about how we refer to objects in modal space.

5. Five-Dimensionalism

Suppose that an elephant is standing calmly in an enclosure at a zoo. Call the elephant "Elfie." When a blind person touches Elfie's trunk, she is touching Elfie. She is touching Elfie by touching its trunk. When another blind person touches Elfie's belly, he is touching Elfie. He is touching Elfie by touching its belly. The two blind people touch one and the same elephant, Elfie, in two different places. When the first person says, "The animal I am touching is like a snake," she is speaking of Elfie and saying of Elfie that it is like a snake. When the second person says, "The animal I am touching is like a wall," he is speaking of Elfie and saying of Elfie that it is like a wall.

Two points should be noted for our purposes with this version of a familiar Indian parable. First, the two blind people are touching two different parts of Elfie. They are perceiving (by touch) two different objects, a trunk and a belly. There is certainly a sense in which they are perceiving one and the same object, namely, Elfie. But it is equally certainly the case that they are perceiving different parts of Elfie, and it is this fact that we should note well for our purposes. Second, the two blind people do not say, "What I am touching is like …" Instead they use the concept *animal* and say, "The animal I am touching is like …" This gives unity to the subject matter of their discourse. Both speakers are speaking of one and the same animal, viz., Elfie. Elfie has two different parts, and the two blind people are touching them, but these parts are not animals; they are animal parts. Thus, the two people are perceiving two different objects (as well as perceiving one animal) and speaking of one and the same common object (the animal), where the two objects are different parts of the common object.

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[9] More cautiously put, if the original matter of Alfred at w_0 has some relevance to answering the question whether w_2 is R to w_1, it is not clear what it is.

I propose to extend this picture to our consideration of modal space. At w_0, a person p_0 perceives a particular alpha, Alfred, points to it, and says, "The alpha I am pointing to originated from 1-2-3, but could have originated from 4-2-3." At w_1, a person p_1 perceives the same particular alpha, Alfred, points to it, and says, "The alpha I am pointing to originated from 4-2-3, but could have originated from 4-5-3." In the elephant parable, the two blind people are separated from each other in physical space. They are at two different locations and are perceiving two different spatial parts Elfie has at these locations. Here, p_0 and p_1 are separated from each other in modal space. They are at two different worlds and perceiving two different worldly parts (world stages) which Alfred has at these worlds. This presupposes, of course, that Alfred is extended in modal space, having different worldly parts at different worlds, in a way analogous to the way Elfie is extended in physical space, having different spatial parts at different locations. This picture may be said to be a picture of *five-dimensionalism*. It incorporates three physical spatial dimensions, one temporal dimension, and one modal dimension.[10] Embrace this five-dimensionalist way of understanding Chandler's scenario, and Axiom 4 is saved. Or so I claim.

It should be noted that even though p_0 at w_0 and p_1 at w_1 perceive the same particular alpha, namely Alfred, this does not preclude p_1's also perceiving a different alpha (or p_0's also perceiving a different alpha, for that matter). The worldly part Alfred has at w_1 may be a worldly part (w_1-stage) of another alpha; two different alphas may share one and the same w_1-stage. If this is indeed the case, then when p_1 says, "The alpha I am pointing to originated …, but could have originated …," p_1 is pointing to a particular w_1-stage shared by two different alphas. So p_1's use of the definite description "the alpha I am pointing to" has no unique denotation, hence no denotation (as "the" implies uniqueness)—unless some additional restriction on the allowable denotation is implicitly assumed. I shall propose to take this idea seriously in my attempt to preserve S4.

6. Overlap and Reference Shift

I shall now extend the five-dimensionalist recasting of Chandler's scenario and give my proposal. First, here is my recasting of Chandler's scenario.

Alfred has different worldly parts at w_0 and at w_1. Call these parts *Alfred-at-w_0* and *Alfred-at-w_1*, respectively. Alfred-at-w_0 exists at w_0 and at no other world, while Alfred-at-w_1 exists at w_1 and at no other world. When p_0 points to an alpha at w_0, p_0 points to Alfred by pointing to Alfred-at-w_0, and thereby succeeds in speaking of Alfred when she says, "The alpha I am pointing to originated from 1-2-3 but could have originated from 4-2-3." She speaks truthfully, for at some world, viz., w_1, accessible from w_0, Alfred originated from 4-2-3. Alfred originated from 4-2-3 at w_1 by having a worldly

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[10] This assumes that the modal is just one-dimensional. This is nothing more than a simplifying assumption for the sake of discussion. The modal should probably be considered multi-dimensional. It also assumes that what exists at a world is four-dimensional. Those who regard the number of dimensions of what exists at a world to be less than four could add a modal dimension as an additional dimension but would resist calling the result "five-dimensionalism."

part at w_1 which originated from 4-2-3. When p_1 points to an alpha at w_1, p_1 points to Alfred by pointing to Alfred-at-w_1, and thereby succeeds in speaking of Alfred when he says, "The alpha I am pointing to originated from 4-2-3 but could have originated from 4-5-3." He speaks truthfully, for at some world, viz., w_2, accessible from w_1, Alfred originated from 4-5-3. Alfred originated from 4-5-3 at w_2 by having a worldly part at w_2 which originated from 4-5-3. Here the accessibility relation remains constant; w_2 is accessible from w_1 in the same sense in which w_1 is accessible from w_0. The same accessibility relation is in question for both pairs of worlds. So there is no shift from one variety of metaphysical possibility to another variety of metaphysical possibility, as is the case in Murray and Wilson's proposal. One and the same accessibility relation holds between w_2 and w_1, and between w_1 and w_0. But at w_0 Alfred could not possibly have originated from 4-5-3. Therefore, accessibility is not transitive.

In order to block this five-dimensionalistically recast argument by Chandler, let us return to the elephant parable and modify it a little.

Suppose that instead of one elephant, two elephants—Elphie$_0$ and Elphie$_1$—are standing calmly. Suppose further that Elphie$_0$ and Elphie$_1$ are Siamese twins joined at the belly. When the first blind person touches Elphie$_0$, she touches its trunk but not the trunk of Elphie$_1$, so that when she says, "The animal I am touching is like a snake," she is speaking of Elphie$_0$, and not Elphie$_1$. When the second blind person touches the elephant belly, he is touching the common part of Elphie$_0$ and Elphie$_1$, so that when he says, "The animal I am touching is like a wall," he is not speaking of Elphie$_0$ to the exclusion of Elphie$_1$, or of Elphie$_1$ to the exclusion of Elphie$_0$. The definite description "the animal I am touching" in his mouth fails to denote a unique animal.[11] If it denotes at all, it denotes ambiguously. Or we may say that it denotes a unique animal equivocally. It is open to two different but equally good interpretations, according to each of which it denotes a unique animal. When the story is told in such a way that Elphie$_0$ is introduced first and then Elphie$_1$ is mentioned only later, it is natural and tempting—because of the overlap—to see the second blind person as touching the slightly more familiar elephant, viz., Elphie$_0$, so that we end up being drawn to the interpretation of his definite description "the animal I am touching" as denoting Elphie$_0$.

With this picture of Siamese twin elephants clear in mind, let us return to Alfred. What transpires in the situation concerning Alfred is analogous to that concerning the Siamese twin elephants. When we say that p_0 truthfully says of Alfred that it could have originated from 4-2-3, we refer by our use of the name "Alfred" to a certain modally extended alpha which originated from 1-2-3 at w_0. So far so good. But just as the second blind person's term "the animal I am touching" is equivocal, our term "Alfred" is equivocal as we use it to describe p_1.[12] When we describe p_1 as saying of Alfred that it could have originated from 4-5-3, our use of "Alfred" is open to two

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[11] Assuming that the two Siamese elephants together as a whole are not one animal.

[12] "Alfred" is also equivocal as we use it to describe what p_0 says, as will become clear once the entire picture is in view. But dialectically we need here to focus on "Alfred" as we use it to describe what p_1 says.

different and equally good ways of understanding. When understood one way, it refers to one alpha, and when understood the other way, it refers to another, different alpha.

At w_0 Alfred could have originated from 4-2-3. So, we say, there is a world w_1 accessible from w_0 such that at w_1 Alfred originated from 4-2-3. When we say this, the term "Alfred," as it occurs in our clause "at w_1 Alfred originated from 4-2-3," is quite naturally understood to be coreferential with "Alfred" as it occurs in the preceding sentence "At w_0 Alfred could have originated from 4-2-3." This corresponds to the interpretation of "the animal I am touching" as uttered by the second blind person according to which it denotes the same animal as it does when uttered by the first blind person, that is, it denotes Elphie$_0$.

But after noting that at w_1 Alfred originated from 4-2-3, we also say that at w_1 Alfred could have originated from 4-5-3, and therefore at some world w_2, accessible from w_1, Alfred originated from 4-5-3. When we say this, it is more natural and charitable to understand the term "Alfred" occurring in the clause "at … w_2 … Alfred originated from 4-5-3" as referring to another, different alpha, of which it is true to say that it has a worldly part that originated from 4-5-3. This corresponds to the interpretation of "the animal I am touching," as uttered by the second blind person, according to which it denotes Elphie$_1$.[13] We are using the name "Alfred" to refer to one alpha and then to refer to another alpha. This is the shift that saves transitivity of accessibility.

Natural and charitable as they are, these shifty readings of our use of the name "Alfred" are not inevitable, and one may insist on the non-shifty reading according to which all of our uses of "Alfred" refer to just one object. Is such a reading compatible with five-dimensionalism? Does it preserve transitivity of accessibility?

It is certainly compatible with five-dimensionalism, for one may say with legitimacy that the one object "Alfred" refers to in all of our uses is a five-dimensionally spread-out object whose parts include the respective worldly parts in question at the worlds, w_0, w_1, and w_2. If such an object is an alpha, then accessibility is not transitive, as shown by Chandler's argument. So in order to resist the argument, we need to say that such an object is not an alpha. And here again the elephant analogy helps us. We can say that such an object is not an alpha any more than the entire Siamese-twin elephants as a whole are an elephant. We have two elephants, not one. Likewise in the case of "Alfred," we have two alphas,[14] not one. The whole object consisting of the two elephants is not itself an elephant, but it is still something. But since it is not an elephant, the condition of origin applicable to elephants need not apply to it. Similarly, the five-dimensionally spread-out object having the worldly parts at w_0, w_1, and w_2 is something. But since it is not an alpha, the condition of origin applicable to alphas need not apply to it. In other words, if we insist on using "Alfred" to refer to a

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[13] I have not supplied a detailed narrative about Elphie$_1$ which would make the correspondence more vivid, but it should not be difficult to do so.

[14] *At least* two alphas. We really have more than two alphas, but what is important is that we have more than one, that is, we do not have a uniquely determined alpha.

single five-dimensionally spread-out object whose parts include the relevant worldly parts at w_0, w_1, and w_2, then what we refer to by "Alfred" is not an alpha. So, Chandler's argument will lose its foothold.

Suppose that we disambiguate the name "Alfred" and call the alpha (the original alpha) we refer to when considering w_0 *Alfred$_0$*, and the alpha we refer to when considering w_1 *Alfred$_1$*. Then Chandler's argument may be formulated as follows:

P$_a$: Alfred$_0$ originated from 4-5-3.
P$_b$: Alfred$_1$ originated from 4-5-3.
1". P$_a$ is not true at any world accessible from w_0.
2". P$_a$ is not possibly true at w_0.
3". P$_b$ is true at w_2, and w_2 is accessible from w_1, which is accessible from w_0.
4". P$_a$ is possibly possibly true at w_0.
5". Some proposition, viz., P$_a$, is not possibly true but possibly possibly true, at w_0.
6". Accessibility is not transitive, i.e., Axiom 4 is false.

3" does not yield 4", which is needed, along with 2", to yield 5". 3" does yield "P$_b$ is possibly possibly true at w_0," but this does not help us reach 5", for 2" concerns not P$_b$ but P$_a$. If we replaced 2" with "P$_b$ is not possibly true at w_0," then the argument's validity would be restored. But nothing in Chandler's scenario shows that P$_b$ is false at every world accessible from w_0, so such a replacement is unsupported.

7. Impossible Worlds

Still, it is true to say that at w_0 Alfred$_0$ could not have originated from 4-5-3, i.e., at w_0 it is impossible that Alfred$_0$ originated from 4-5-3. And according to Chandler, Alfred$_0$ indeed exists and originated from 4-5-3 at w_2. So assuming that w_0 is the actual world, w_2 is an impossible world, according to Chandler. So, Chandler should be happy to place Alfred$_0$ at an impossible world, and Salmon quite explicitly does so and emphasizes the impossibility of the world w_2. How do I respond to this stance on an impossible world by Chandler-Salmon?

One way to respond is to downplay the significance of impossible worlds. This is the way of Murray and Wilson.[15] But we need not follow them. We can perfectly well go along with taking impossible worlds seriously. Assuming that w_0 is the actual world, according to Chandler-Salmon, w_2 is an impossible world, for w_2 is not accessible from w_0. Although I do not want to downplay the significance of impossible worlds, I think it is a mistake to regard w_2 as inaccessible from w_0. At w_2, an alpha indeed originated form 4-5-3 but it is not Alfred$_0$. Alfred$_0$ does not exist at w_2. The alpha which exists and originated from 4-5-3 at w_2 is Alfred$_1$.

At w_0 Alfred$_0$ originated from 1-2-3 and could have originated from 4-2-3. So at some accessible world, w_1, Alfred$_0$ originated from 4-2-3. A different alpha sneaks into the scene at this point, namely, Alfred$_1$. At w_1, Alfred$_1$, like Alfred$_0$, originated from 4-2-3 and could have originated from 1-2-3, but unlike Alfred$_0$, it also could have originated from 4-5-3. Conflating the two alphas is at the core of Chandler's error, as we

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[15] Murray and Wilson say that Salmon's mention of an impossible world is a distraction.

have observed. But there is a third alpha, which just sneaked into the picture as we spoke of possible origination from 4-5-3 at w_1. At w_2, which is accessible from w_1, $Alfred_1$ originated from 4-5-3. That is, $Alfred_1$'s w_2-stage originated from 4-5-3. Just as $Alfred_0$ overlaps $Alfred_1$ at w_1, $Alfred_1$ overlaps another, third alpha at w_2. Interestingly, Chandler, who does not show awareness of the second alpha ($Alfred_1$), explicitly mentions what is effectively this third alpha; he calls it *Bernard*.[16] Does the introduction of Bernard force an impossible world upon us? No, it does not. Let us see why not.

At w_0 it is possible that $Alfred_0$ never exists but parts 4-5-3 do. So at some accessible world, call it w_3, $Alfred_0$ does not exist but 4-5-3 do. Suppose further that at w_3 an alpha originated from 4-5-3. This alpha is Bernard. Since w_3 is accessible from w_0, and 4-5-3 and 1-2-3 have only 3 in common, Bernard is not $Alfred_0$. $Alfred_0$ has no worldly part at w_3 or at w_2. Bernard has a worldly part at w_1, which is the w_1-stage of an alpha originating from 4-2-3. But of course, that w_1-stage is the w_1-stage of $Alfred_1$ and of $Alfred_0$ as well; Bernard overlaps $Alfred_1$ and $Alfred_0$ at w_1. Bernard overlaps $Alfred_1$, but not $Alfred_0$, at w_2.

I simply identify w_3 with w_2. Nothing forces this identification; it is possible to maintain that $Alfred_1$ has no worldly part at w_3. But at the same time, nothing forbids the identification, and the identification simplifies the picture. When someone says, having in mind the common w_1-stage in question, "Alfred could have originated from 4-5-3," by the name "Alfred" she either refers to $Alfred_1$ or Bernard and says something true, or else refers to $Alfred_0$ and says something false. This is how I block Chandler's argument.

Still, I agree with Chandler-Salmon that $Alfred_0$ exists and originated from 4-5-3 at some impossible world. It is just that I deny that w_2 ($=w_3$) is such a world. Let w_{20} be such a world. For Chandler-Salmon's purposes, w_{20} is w_2. Chandler-Salmon's argument is in effect that since w_{20} ($= w_2$) is accessible from w_1, w_1 is accessible from w_0, and w_{20} ($= w_2$) is inaccessible from w_0, accessibility is not transitive. I agree that w_1 is accessible from w_0 and that w_{20} is inaccessible from w_0, but I deny that w_{20} is accessible from w_1. To think that w_{20} is accessible from w_1 is to conflate $Alfred_0$ with $Alfred_1$.[17]

It is important not to forget that even though an alpha originated from 4-5-3 at w_2, it is a different alpha from $Alfred_0$; it is $Alfred_1$. $Alfred_0$ originated from 4-5-3 at w_{20} instead. The two worlds, w_2 and w_{20}, may well be qualitatively indistinguishable. Even so, they are distinguishable with respect to the identity of the alpha originating from 4-5-3; they are two distinct worlds.[18] Moreover, they are differently related to

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[16] Chandler credits Robert Stalnaker for suggesting the Bernard example.

[17] Or worse, with Bernard.

[18] This raises an interesting issue of haecceitism, which is the claim that for any possible worlds w and w', if w and w' are qualitatively indistinguishable, then they are indistinguishable *simpliciter* (see Lewis 1986). In particular, two possible worlds agreeing in all matters qualitative agree in all matters *de re*. The pair of worlds $\{w_2, w_{20}\}$ might look like a counterexample to haecceitism in this sense. But they are not; for a genuine counterexample needs to be a pair of possible worlds, and w_{20} is not a possible world (assuming that w_0 is the actual world).

other worlds: w_2 is accessible from w_0 and from w_1, but w_{20} is not accessible from w_0 or from w_1.

My position is that no impossible world is accessible from a possible world. Assuming that w_0 is the actual world, "possible world" means "world accessible from w_0" and "impossible world" means "world inaccessible from w_0." Moreover, I am assuming with Chandler-Salmon that w_1 is accessible from w_0 and w_{20} is inaccessible from w_0, and claiming that w_{20} is inaccessible from w_1. This claim of mine then amounts to the claim that even though Alfred$_0$ could possibly have originated from 4-2-3, if it had so originated, it would not be possible for Alfred$_0$ to have originated from 4-5-3. But in making this claim, am I not flouting origin essentialism about alphas? Am I not simply denying the principle that for any alpha, if it originated from x-y-z, then it is possible for it to have originated from x-v-z, where v ≠ y?

Someone might respond on my behalf by making a distinction between two principles of origin essentialism and affirming one of them, while denying the other. The two principles to be distinguished are as follows:

(E1) For any alpha, if it originated from x-y-z, then it is possible for it to have originated from three parts two of which are x, y, or z.

(E2) For any alpha, if it could have originated from x-y-z, then it could have been possible to have originated from three parts two of which are x, y, or z.

My hypothetical spokesperson might affirm (E1) and deny (E2).[19] In the possible-worlds framework, this asymmetric treatment of the two principles amounts to privileging the actual world. If an alpha originated from x-y-z at the actual world, it originated from x-v-z at some world accessible from the actual world; but if an alpha originated from x-y-z at a non-actual world w, there might or might not be a world accessible from w at which it originated from x-v-z. Such privileging of actuality is in concert with the fact, of which Saul Kripke famously reminded us,[20] that we standardly discuss non-actual possibilities by starting with actually existing objects and stipulatively considering non-actual possibilities concerning *them*.

Suppose that w_0 is the actual world and that at w_0 you point to an alpha which originated from 1-2-3 and say truthfully, "This is Alfred and Alfred is an alpha that could have originated from 4-2-3." Suppose also that at w_1 someone like you points to an alpha which originated from 4-2-3 and say truthfully, "This is Alfred and Alfred is an alpha that could have originated from 4-5-3." These statements are both true, for you refer to Alfred$_0$ by "Alfred" at w_0 and the person in question at w_1 refers to Alfred$_1$ by "Alfred" at w_1. This generalizes to yield the claim that *standardly* no two people attaching a name to an alpha by ostension at two different worlds are speaking of the same five-dimensional alpha; the alphas they are speaking of may well overlap each other extensively, but their five-dimensional extents are not exactly the same. (If, instead of alphas, we have objects which are governed by vague origin conditions for

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[19] (E1) is a metaphysical claim of origin essentialism. I am assuming that all parties involved in the discussion of the Chandler-Salmon argument regard it as a necessary truth (at least for the sake of argument), hence a legitimate starting point in modal reasoning.

[20] Kripke 1980: 44.

numerical identity, then we may be able to avoid this and ensure the exact sameness of the extents by having an account of vagueness concerning the number of parts essential for origin such that the non-overlapping worldly parts of the objects lie well within the penumbra allowed by the vagueness.)

This, of course, does not mean that when speaking at w_0, you cannot speak of a particular alpha as having originated from 4-5-3 at some world other than w_0. You may perfectly well point to an alpha in front of you and say, "This is Alfred and Alfred is an alpha which could have originated from 4-2-3, and if it had originated from 4-2-3, then it would have been possible for it to have originated from 4-5-3." You may perfectly well be speaking of $Alfred_0$ throughout your remark; the subject matter of your speech may well remain to be uniquely $Alfred_0$. But if it does so remain, then the second conjunct of your statement is false; it is false that if $Alfred_0$ had originated from 4-2-3, then it would have been possible for $Alfred_0$ to have originated from 4-5-3. At w_1 $Alfred_0$ has a worldly part which originated from 4-2-3, and it is also a worldly part of $Alfred_1$. And it is $Alfred_1$, not $Alfred_0$, that could at w_1 possibly have originated from 4-5-3.

This seems to work as a defense of my proposal. Should I accept (E1), reject (E2), and let my hypothetical spokesperson speak on my behalf then? I am afraid not. Privileging of actuality is an interesting idea and the spirit in which it is proposed will prove to be productive, as we shall see shortly. But if we wish to preserve standard quantified modal logic (SQML)—and I do, as I endeavor to defend Axiom 4, and also Axiom 5 later—then we cannot reject (E2) while accepting (E1); for in SQML, (E1) entails (E2). Let "v" be a restricted variable ranging just over alphas, and let "Fv" and "Gv" mean "v originated from x-y-z" and "v originated from three parts two of which are *x*, *y*, or *z*," respectively. Then (E2) is derived from (E1), as follows:

$\forall v(Fv \rightarrow \Diamond Gv)$ (E1)
$\forall v(Fv \rightarrow \Diamond Gv)$ by the rule of necessitation
$\forall v \ (Fv \rightarrow \Diamond Gv)$ by the Converse Barcan Formula
$\forall v(\Diamond Fv \rightarrow \Diamond\Diamond Gv)$ (E2) by the valid schema $(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$[21]

This means that within SQML we cannot privilege actuality by means of a distinction between (E1) and (E2), or anything that entails the distinction. This is where the crucial idea of reference shift proves useful.

Some might cast doubt on the first step in the above argument, from (E1) to its necessitation, on the ground that (E1) is not a theorem of SQML. Such a doubt is allayed when we note that the argument starts with (E1) not because (E1) is a theorem of SQML (it certainly is not) but because, as indicated in footnote 19, all parties accept it as a metaphysical claim which does not just happen to be true but is necessarily true. Since Chandler takes himself to be arguing in effect against SQML as he argues under this assumption, it is permissible for me to make the same assumption.

Reference shifts from world to world as we consider whether what Chandler calls "Alfred" had or could have had a certain origin. Along with this reference shift, we

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[21] I owe this proof to Alessandro Torza.

should shift the way we apply the idea of essentiality of origin. In fact, without the latter shift, the reference shift alone would be rather pointless for my purposes. When we consider Chandler's alpha at w_0 and say, "At w_0 Alfred$_0$ could have originated from 4-2-3," we are applying the principle of origin essentialism correctly, but if we moved our consideration to w_1 and said, "At w_1 Alfred$_0$ could have originated from 4-5-3," we would not be applying the principle of origin essentialism correctly. When we shift our talk to w_1 with Chandler, we shift our reference of the name "Alfred." This is my claim of reference shift. Resorting to such reference shift would be moot unless application of the principle of origin essentialism is also adjusted so that the new referent is the subject matter when we say that *it* could have originated from 4-5-3.

On my proposal of reference shift, we refer to different alphas when considering different worlds in accordance with Chandler's scenario. The correct way to apply the principle is to say that at w_0 Alfred$_0$ could have originated from 4-2-3, that at w_1 Alfred$_0$ could have originated from 1-2-3, that at w_1 Alfred$_1$ could have originated from 4-5-3 or from 1-2-3, that at w_1 Bernard could have originated from 4-5-3, and that at w_2 Bernard could have originated from 4-2-3. It is an incorrect application of the principle to say that at w_1 Alfred$_0$ could have originated from 4-5-3, or that at w_1 Bernard could have originated from 1-2-3.

The principle of origin essentialism for alphas allows one-third deviation in origin away from, or toward, the *modal center* of the alpha in question, but in no other direction. The (modal) center of Alfred$_0$ is its w_0-stage, the center of Alfred$_1$ is its w_1-stage, and the center of Bernard is its w_2-stage. Since Alfred$_0$'s center is its w_0-stage, which originated from 1-2-3, at w_0 Alfred$_0$ could have originated from, for example, 4-2-3, 7-2-3, 1-10-3, 1-11-3, 1-2-13, or 1-2-14 (away from the center), and at w_1 Alfred$_0$ could have originated from 1-2-3 (toward the center). But at w_0 Alfred$_0$ could not have originated from, for example, 1-5-6, 4-2-6, or 4-5-3 (too far away from the center). At w_1, where Alfred$_0$ is not centered, Alfred$_0$ could have originated from, for example, 1-2-6 or 1-5-3 (to the center then away from it). Thus, assuming that Alfred$_0$ originated from 1-2-6 at w_7, w_7 is accessible from w_1, even though at w_1 Alfred$_0$ originated from matter which is two-thirds different from the matter from which it originated at w_7.

It is natural that the appearance to the contrary is created and we are tempted to answer "No" when we ask ourselves the question, "At w_1 could Alfred$_0$ have originated from 1-2-6?" This is because when we fix our attention on possibilities at w_1, we are naturally led to assume that we are speaking of an alpha centered at w_1. But since we are in fact speaking of Alfred$_0$ and Alfred$_0$ is centered at w_0, not at w_1, the answer we are tempted to give is the wrong answer.

Alfred$_0$ Alfred$_1$ Bernard
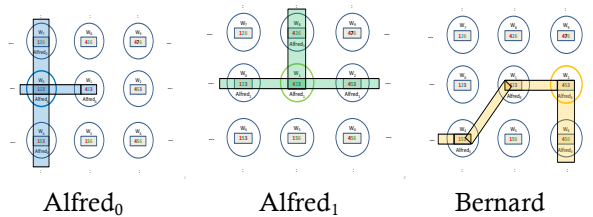
In general, assuming that Alfred$_k$'s center is its w_k-stage, which originated from x-y-z, at w_k Alfred$_k$ could have originated from matter consisting of at least two of x, y, and z (away from the center) but not from matter consisting of no more than one of x, y, and z; and at w_{k+1}—where Alfred$_k$'s origin differs from x-y-z by exactly one part, say, x-y-v—Alfred$_k$ could have originated from x-y-z (toward the center) or from u-y-z or x-u-z, where u is neither x nor y (to the center then away from it), but not from more different matter than these. Thus the principle of origin essentialism should say:

> (OE) For every alpha, it is necessary that it originated from matter that is at most one-third different from the matter its center originated from (and it is possible that it originated from matter that is only one-third different from the matter its center originated from).

Alphas are artificially well-behaved objects. Ordinary objects are much more complicated. Their origin has many more parts than three, and the principle of origin essentialism for them are much harder to formulate. Still, the basic idea applies to them just as well. Each object is a five-dimensional object with a center, which consists of many world-stages instead of just one. A clear line (like "one-third" for alphas) cannot be drawn, and the issue of vagueness needs to be faced squarely. But these are mere complications, rather than fundamentally different considerations that change the shape of the discussion.

8. Euclidean-ness

Chandler's thought experiment is easily adaptable to produce an argument against the characteristic axiom of S5, viz., Axiom 5: $\Diamond A \rightarrow \Box\Diamond A$. This axiom requires that the accessibility relation be euclidean, that is, if two worlds are accessible from a common world, then they are mutually accessible. In the middle diagram above, Alfred$_1$ originated from 1-2-3 at w_0, from 4-2-6 at w_8, and from 4-2-3 at w_1. So, w_0 and w_8 are both accessible from w_1, but Alfred$_1$'s origin at w_0 and Alfred$_1$'s origin at w_8 have only one common part; so neither of w_0 and w_8 is accessible from the other, someone might say. Therefore, it might be concluded, accessibility is not euclidean, hence Axiom 5 is false.[22]

This objection against Axiom 5 is answerable by resorting to the idea of the center of a modally extended object and using the principle of origin essentialism (OE). Alfred$_1$ is centered at w_1, and Alfred$_1$'s w_0-stage and w_8-stage both originated from matter having two common parts with the matter from which Alfred$_1$'s w_1-stage originated. Thus, according to (OE), Alfred$_1$ at w_0 and Alfred$_1$ at w_8 do not flout origin essentialism. We are assuming that nothing else stands in the way of mutual accessibility between w_0 and w_8. Therefore, w_0 and w_8 are accessible from each other.

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[22] Since w_0 is the (presumed) actual world, this consideration gives us another reason why distinguishing (E1) from (E2) does not help. I owe this point to Axel Barceló.

9. Obstinate Essentialism

Recall that obstinate essentialism says that any object that originated from matter consisting of certain parts necessarily originated from matter consisting of exactly those parts. So, according to obstinate essentialism, $Alfred_0$ necessarily originated from 1-2-3, that is, for any possible world w, an alpha originating from any matter other than 1-2-3 at w, is not $Alfred_0$. Some philosophers may find this position appealing. My proposal of reference shift explains the apparent appeal of obstinate essentialism without endorsing it.

Chandler's scenario starts with an alpha originating from 1-2-3 at w_0; that alpha is $Alfred_0$. According to my proposal of reference shift, when we consider an alpha at w_1 originating from 4-2-3, we are naturally and most likely referring to $Alfred_1$, not $Alfred_0$; that is why it is natural and most likely true to say then, "At w_1 Alfred could have originated from 4-5-3." When we consider the relevant alpha at yet another relevant world, w_2, we are naturally and most likely referring to Bernard, not $Alfred_0$ or $Alfred_1$. (The relevant alpha at a relevant world is an alpha that is different from the previous alpha in origin by one third and existing at a world minimally different from the previous world.) These are all different alphas, hence the impression is created that no alpha survives the minimal amount of change in origin across worlds.

At the same time, what is overlooked by the obstinate essentialist is that since $Alfred_0$ and $Alfred_1$ overlap at w_1, having the common w_1-stage, which originated from 4-2-3, we (at w_0) can refer to $Alfred_0$ rather than $Alfred_1$ by pointing (with the mind's finger) to that w_1-stage and considering it in conjunction with considering what exists at w_0, in particular, the w_0-stage of an alpha. When we do so, we speak truthfully by saying, "Alfred originated from 1-2-3 at w_0 but originated from 4-2-3 at w_1, so it could have originated from matter slightly different from the matter it actually originated from." By contrast, if we point to the same w_1-stage and consider it not in conjunction with considering the w_0-stage of an alpha existing at w_0, but afresh as the starting point of discussing origin essentialism, then we may well be able to speak truthfully by saying, "Alfred originated from 4-2-3 at w_1, so it could have originated from 4-5-3," referring to $Alfred_1$ by "Alfred."[23]

10. Alternative Modal Space

The thesis I defend says that the accessibility relation underlying metaphysical possibility is transitive in modal space. Chandler-Salmon say that it is not transitive. So I say that they get modal space wrong. At the same time, I think that they get something right. Let me explain.

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[23] The question which modally extended object we succeed in referring to via a particular worldly stage is not an easy question to answer. When we point to the portion of the body shared by two Siamese twins while intending to refer to one of the twins as opposed to the other, what determines which twin we succeed in referring to? Is our intention alone sufficient for the determination? Or do other contextual factors figure somehow, and if so, what factors and how? The modal case is no easier than the Siamese twin case to settle.

World indexing works well for evaluations of truth-values of ordinary statements about ordinary objects like bicycles, boxes, branches, and bears, but not for evaluations of truth-values of some statements about worlds. Statements of logical properties, like transitivity, concerning the accessibility relation are statements about worlds, as both *relata* of accessibility are worlds. Some relations between worlds may be said to hold or fail to hold only relative to a world. Take closeness, for example; a world may be closer than another world in the sense that the first world resembles a certain given world more than the second world does. In such a case, we may regard the dyadic closer-than relation as holding between two worlds from the point of view of the given world, that is, the dyadic relation as holding between two worlds relative to a world. But accessibility is not like that. A world is accessible from another world (or not), independently of relativization to a given world. What particular accessibility relation is in question in a given discourse is, of course, relative to what is relevant to the discourse; it may be metaphysical accessibility in one discourse, logical accessibility in another discourse, and nomological accessibility in yet another. But once a particular accessibility relation is determined in a given discourse, which world bears that relation to which world is not a matter relative to a world. So whether the particular accessibility relation underlying metaphysical possibility is transitive is not a matter relative to a world. But it is still relative, and this allows room for something that Chandler-Salmon get right.

Whether a given relation has or lacks a given logical property, like transitivity, is determined globally within the entire realm that comprises eligible *relata*. When the given relation is the accessibility relation underlying metaphysical possibility, the relevant realm is a realm comprising worlds. If for some triple of worlds w, w', and w'' in the realm, accessibility holds between w and w', and between w' and w'', but not between w and w'', then it is correct to say relative to the realm that accessibility is not transitive; otherwise, it is correct to say relative to the realm that it is transitive. I think that since there is no such triple of worlds relative to a certain realm we (implicitly) have in mind when discussing Chandler-Salmon's challenge to S4, accessibility is transitive relative to the realm. The realm in question is the realm comprising all the worlds standardly considered when *local* metaphysical possibility is in question, viz., *local modal space,* and this is the exact sense in which we say that accessibility is transitive *in modal space*.[24]

Now, when Chandler-Salmon maintain that accessibility is not transitive, I understand them as maintaining that accessibility is not transitive in modal space. Since, in my opinion, accessibility is transitive in modal space, I say that Chandler-Salmon are wrong. But, in their arguments Chandler-Salmon do successfully describe some realm. They simply mistake it for modal space and end up maintaining a falsity. But

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[24] The adjective "local" modifies the noun phrase "modal space" in a way parallel to the way in which "actual" modifies "world." If the actual world is the default world of our discourse, the local modal space is the default modal space of our discourse. Indeed, the parallel is so striking that some of us might even be tempted to use "actual" instead of "local" to modify "modal space."

their description applies correctly to that realm, all the same. And that realm is just like modal space but it is alternative to it. It is a realm in which for some triple of worlds, w, w', and w'', accessibility holds between w and w', and between w' and w'', but not between w and w''. This *alternative (non-local) modal space* does indeed comprise a world (w) at which Alfred, which actually (at w'') originated from 1-2-3, originated from 4-5-3, while remaining an alpha. Chandler-Salmon say that such a world is an impossible world, and I agree. Chandler-Salmon conclude from this—assuming the relevant background information about w, w', and w''—that accessibility is not transitive, that is, accessibility is not transitive in modal space. I, by contrast, say that Chandler-Salmon change the subject. The impossible world w does not belong to modal space, viz., *the local modal space*. It belongs to some alternative modal space, and in that alternative modal space, accessibility fails to be transitive. But that alternative modal space is not the modal space in question—our *local modal space*—but comprises a metaphysically impossible world which robs accessibility of a logical property it in fact has in our local modal space.

11. Speculation

There are many modal spaces, and each of them comprises worlds. Our actual world, assumed to be w_0, exists in our local modal space—call it s_0—and also in other modal spaces. Many other worlds also exist in s_0 and in other modal spaces. Every world exists in some modal space, but probably not in every modal space. Let us say that w_0 *locally$_s$* exists in s_0 and non-locally$_s$ but *alternatively$_s$* exists in another modal space. Locality$_s$ and alternativeness$_s$ pertain to modal spaces in a way parallel to the way in which actuality and mere possibility pertain to worlds. The world w_1 is accessible from w_0 in s_0 (w_1 is locally$_s$ accessible from w_0), but it is not accessible from w_0 in some other modal space (it is alternatively$_s$ not accessible from w_0). Accessibility is transitive in s_0 (accessibility is locally$_s$ transitive), but it is not in some other modal space (it is alternatively$_s$ not transitive). The worlds, w_0, w_1, and w_2, locally$_s$ exist, w_2 is locally$_s$ accessible from w_1, w_1 is locally$_s$ accessible from w_0, and w_2 is locally$_s$ accessible from w_0. The worlds w_0 and w_1 also alternatively$_s$ exist, along with w_{20}, in some alternative modal space, and in that alternative modal space, w_{20} is alternatively$_s$ accessible from w_1, w_1 is alternatively$_s$ accessible from w_0, and w_{20} is alternatively$_s$ not accessible from w_0.[25] It is such an alternative modal space that Chandler-Salmon inadvertently end up describing.

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[25] When I say this, I do not mean that in that alternative modal space, w_{20} is not but could be accessible from w_1, w_1 is not but could be accessible from w_0, and w_{20} is not but could be inaccessible from w_0. Rather, I mean that in that alternative modal space, w_{20} is accessible from w_1, w_1 is accessible from w_0, and w_{20} is inaccessible from w_0. The point of the adverb "alternatively$_s$" is to distinguish these predications of accessibility and inaccessibility from predications within local modal space, which in contrast are to be marked by the adverb "locally$_s$." There is more to be said about this and what I call *modal tense*; for the basic idea of modal tense, see Yagisawa 2010, chapter 5.

When we say that accessibility is transitive, what we usually mean is that accessibility is locally$_s$ transitive, just as when we say that the universe is expanding, what we usually mean is that the universe is actually expanding. When we say that since it is possible that Bernard originated from 4-5-3, Bernard originated from 4-5-3 at a possible world, we should mean that Bernard locally$_s$ originated from 4-5-3 at a possible world, just as when we say that Caesar crossed the Rubicon, we should mean that Caesar actually crossed the Rubicon. When, on the other hand, we insist that since it is impossible that Alfred$_0$ originated from 4-5-3, Alfred$_0$ originated from 4-5-3 at an impossible world, we should mean that Alfred$_0$ alternatively$_s$ originated from 4-5-3 in some modal space in which worlds are related in an alternative$_s$ way. I emphasize that when we insist on understanding the statement of the impossibility of Alfred$_0$ having originated from 4-5-3 by locating Alfred$_0$'s having originated from 4-5-3 at an impossible world, our subject matter automatically shifts from the local$_s$ modal space s_0 to some alternative$_s$ modal space, call it s_{20}, comprising the impossible world w_{20}, along with the worlds w_0 and w_1.

Alfred$_0$'s having an impossible origin 4-5-3 is to be located at a world outside s_0, for to speak of Alfred$_0$ having originated from 4-5-3 is to speak of Alfred$_0$ having a worldly part which Alfred$_0$ does not in fact have, i.e., does not have in s_0. Such Alfred$_0$ spreads out in some modal space in a way different from the way Alfred$_0$ spreads out in s_0. Such Alfred$_0$ spreads out in some alternative modal space, s_{20}. The s_{20}-stage of Alfred$_0$ has a different shape (spread) from the s_0-stage of Alfred$_0$. Alfred$_0$ is extended not just five-dimensionally[26] but also six-dimensionally.[27]

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[26] Thus the title of this paper should be taken with a grain of salt. In fact, the number of dimensions of Alfred$_0$'s spread does not stop at six. Alfred$_0$ is not an *n*-dimensional object, for any finite *n*. But this is a further speculative idea to be explored on another occasion.

[27] Shorter versions of this paper were presented at the following conferences in 2014: Veritas Philosophy Conference, Yonsei University, Seoul, South Korea, June 8; Intensional/Hyperintensional, National Autonomous University of Mexico, Mexico City, September 18; Modal Metaphysics: Issues on the (Im)Possible II, Institute of Philosophy of Slovak Academy of Sciences, Bratislava, Slovakia, October 15. I thank the audiences for useful discussion. I am particularly grateful to Axel Barceló and Alessandro Torza for helpful challenges. I also thank the two anonymous referees for *Argumenta*.

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