Propositions as Truthmaker Conditions

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Abstract

Propositions are often aligned with truth-conditions. The view is mistaken, since propositions discriminate where truth conditions do not. Propositions are hyperintensional: they are sensitive to necessarily equivalent differences. I investigate an alternative view on which propositions are truthmaker conditions, understood as sets of possible truthmakers. This requires making metaphysical sense of merely possible states of affairs. The theory that emerges illuminates the semantic phenomena of samesaying, subject matter, and aboutness.

Keywords: Propositions, Truthmakers, Hyperintensionality, Content, Meaning.

1. Introduction

The business of a proposition is to be true or false, depending on how things are. To every proposition corresponds a *truth condition*, displaying how things must be for that proposition's truth. It is natural to take a proposition and its truth condition to be one and the same entity, for that proposition is, by its very nature, true in just those situations set out by its truth condition.

As natural as it is, the view cannot be right, for propositions discriminate where truth conditions do not. A truth condition (as commonly understood) is blind to necessarily equivalent distinctions. Not so for propositions. As the heptasyllabic-happy jargon has it, propositions are *hyperintensional*. A proposition distinguishes between necessarily equivalent situations when they ground its truth in different ways. A proposition's identity goes with the different ways of its being true, and not merely with the different situations in which it is true. Propositions are not truth conditions; they are truth*making* conditions.

Propositions as truthmaker conditions: that is the view I shall articulate and defend. Along the way, I shall ask, just what is a *condition*, and why are propositions not truth conditions? (§2) What does it mean to say that a proposition is a truthmaker condition? (§3) And how can such a view be metaphysically respectable? (§4, §5) The theory that emerges illuminates the phenomena of speakers *saying the same* as one another, but in different ways (§6), and of a statement's *subject matter* and what is about (§7).

2. Truth Conditions and Hyperintensionality

It is common to identify propositions with truth conditions. In classical logic, a truth condition is also a falsity condition. Those who treat propositions as entities need to explain what kind of entity they mean by a *condition*. Suppose we are interested in whether something meets condition *X* in such-and-such situations, and suppose we are interested only in getting a yes-no answer for each such situation. We naturally treat that condition as a function from the situations to the answers, *yes* or *no*. Mathematically, this is a *characteristic function*, and each such function defines a set, containing all and only the input entities for which the function's output is *yes*. It is then both very natural and mathematically elegant to identify the condition itself with that set of situations.

In the case of a truth condition, the input situations are possible worlds and the outputs are *true* or *false*. So we identify a truth condition with a set of possible worlds. Thus, identifying propositions with truth conditions leads us to the view that propositions are sets of possible worlds, defended by Lewis (1986) and Stalnaker (1984; 1976a; 1976b). As a consequence, necessarily equivalent propositions are identical. In particular, there is just one necessarily true proposition (the set of all possible worlds) and one necessarily false proposition (the empty set).

Propositions are not sets of possible worlds, however. One may express necessary truths that are clearly distinct: *that 1=1*, and *that Fermat's Last Theorem is true*, for example. It is not merely that these are distinct sentences. The point is that what they express—what we *say* in uttering them—is so very different in each case.

There are many more ways to make this point. One is via belief: one may believe that 1=1 without believing that Fermat's Last Theorem is true. But I want to avoid this argument from belief, since the identification of propositions as objects of belief is a messy and troublesome business. In saying that David Jones changed the world, I thereby say that David Bowie did, 'Bowie' being the name Jones adopted. But I need not know that, and so need not know (or even believe) that these utterances say the same. I prefer to treat knowledge and belief in a different way (Jago 2014), and to set those concepts aside for present purposes.

Here is another way (not involving belief) of making the distinctness point. The proposition

(1) Lenny is either sleeping or he is not

is about Lenny. By contrast, the proposition

(2) Bertie is either adorable or he is not.

is about Bertie (and not Lenny). *Aboutness* is a relation. So each proposition stands in a relation in which the other does not, and so they are distinct. Yet they are both logical truths; and so they cannot be sets of possible worlds.

Here is yet another way to make the point. Given that Lenny is sleeping, (1) is made true by that very state of affairs. And given that Bertie is adorable, (2) is made true by that, distinct, state of affairs. These propositions stand in the *truthmaking* relation to different states of affairs, and so are distinct entities. So again, they are not sets of possible worlds.

These propositions are more discriminating than truth conditions. A given proposition's nature is not merely to be true or false at a given world, but rather to be made true or made false by specific ways things could be. The way *Lenny*

is, is what makes (1) true, even though it must be true. (Or, more carefully, it is made true by the way Lenny is, if he exists, or by his non-existence, if he does not.) Similarly for Bertie and (2). Propositions are not truth conditions: they are truthmaker conditions.

3. Propositions as Truthmaker Conditions

A *truthmaker condition* is a function from possible entities to *yes* or *no*. A *yes* answer indicates that that entity is a truthmaker (or would be, were it to exist) for the proposition in question. Or, more simply, we can identify this characteristic function with the set it defines, so that truthmaker conditions are sets of possible entities. (Typically, in speaking of truthmakers, I will talk in terms of states of affairs. But I do not want to restrict truthmakers to states of affairs, since any entity x whatsoever is a truthmaker for ⟨x exists⟩.) So, as a first pass, propositions are sets of possible entities (of any kind), and we think of those entities as all the possible truthmakers for that proposition.

We do not want to identify a proposition with the set of its actual truthmakers only. We want the proposition to be a condition on what would make it true, were that entity to exist. So the entities in question will have to include merely possible, as well as actual, entities (just as the sets-of-possible-worlds approach to propositions appeals to merely possible worlds). Just how to make sense of this thought is not at all straightforward. Propositions actually exist. They are sets, and so their members actually exist too. But by definition, merely possible entities do not actually exist. This is a deep metaphysical problem for the approach. I will discuss it in detail in §5.

If propositions are sets of their possible truthmakers, then each of those truthmakers must be a single entity. But propositions can be made true by pluralities. ⟨There are pugs⟩ is made true by each individual pug, but also by pairs of pugs, triples of pugs, and, quite generally, by pug pluralities of any size. Treating ⟨there are pugs⟩ as the set of all possible pugs will ignore these pluralities. We might try to avoid this worry by counting all *subsets* of the proposition in question (as well as all of its individual members) among its truthmakers. This approach will identify ⟨there are at least two pugs⟩ with the smallest set with all possible pug-pairs, pug triples, and so on, as its subsets. That is none other than the set of all possible pugs. So this approach incorrectly identifies ⟨there are at least two pugs⟩ with ⟨there is at least one pug⟩.

A better approach is to capture pluralities through their mereological sum. ⟨There are pugs⟩ is the set of all possible pugs and all possible pug-sums. ⟨There are at least two pugs⟩ is the set of all possible two-or-more-pug-sums. Each member of a proposition is then a *full* (possible) truthmaker for that proposition.

A set of possible truthmakers (so understood) is a truthmaker condition. We might want propositions to encode information about their possible falsemakers also. So understood, propositions are truth-and-falsity-maker conditions. We can identify each of these with a set of possible truthmakers plus a set of possible falsemakers. Let us use the notation $|A|^+$ and $|A|^-$ for these sets, respectively. Call the former single-set notion a *single proposition* and the latter double-set notion a *double proposition*. Then both $|A|^+$ and $|A|^-$ are single propositions, and each double proposition ⟨A⟩ is a pair of single propositions, ($|A|^+$, $|A|^-$). One very nice feature of double propositions is that, if ⟨A⟩ is the pair

$(|A|^+, |A|^-)$, then $\langle\neg A\rangle$ is the pair $(|A|^-, |A|^+)$. This is because $|\neg A|^+ = |A|^-$ and $|\neg A|^- = |A|^+$.

Double propositions are a good way to distinguish between necessarily false propositions. When $\langle A\rangle$ and $\langle B\rangle$ are distinct propositions, we want to distinguish between $\langle A \vee \neg A\rangle$ and $\langle B \vee \neg B\rangle$. (The inability to do this was part of the criticism of sets-of-worlds account in §2.) But by the same token, we should also distinguish between the necessarily false propositions $\langle A \wedge \neg A\rangle$ and $\langle B \wedge \neg B\rangle$. We cannot do this by identifying propositions with sets of possible truthmakers. (So doing would identify both with the empty set.) Double propositions are a neat solution, for $\langle A \wedge \neg A\rangle$ and $\langle B \wedge \neg B\rangle$ differ in their possible falsemakers. A falsemaker for $\langle A \wedge \neg A\rangle$ is whatever truthmakes either $\langle A\rangle$ or $\langle \neg A\rangle$ (or both) (Fine and Jago 2017). In general, such entities truthmake neither $\langle B\rangle$ nor $\langle \neg B\rangle$. So, although $|A \wedge \neg A|^+$ coincides with $|B \wedge \neg B|^+$, in general $|A \wedge \neg A|^-$ will differ from $|B \wedge \neg B|^-$.

Not every set counts as a single proposition, and not every pair of sets counts as a double proposition. A single proposition $\langle A\rangle$ must be *downwards closed* with respect to grounding. If $x \in \langle A\rangle$ and y is a possible ground for x, then $y \in \langle A\rangle$ too. Two points of clarification are in order. First, I am using 'ground' here to mean *full ground*, as opposed to *partial ground*. To illustrate: a conjunction is fully grounded by its conjuncts taken together, and is partially grounded by each of them individually. Second, the closure condition just given makes use of the dyadic notion of grounding: a single entity y as a possible ground for x. As above, pluralities of partial grounds, say x_1 and x_2, are represented as their mereological sum, $x_1 \sqcup x_2$. So we can have $x_1 \sqcup x_2 \in \langle A\rangle$ without $x_1 \in \langle A\rangle$ or $x_2 \in \langle A\rangle$.

We may require that single propositions be upwards-closed with respect to mereological summation: if $x, y \in \langle A\rangle$ then their sum, $x \sqcup y \in \langle A\rangle$. If we do that, then we commit ourselves to impossible entities. If $\langle A\rangle$ has a possible truthmaker x and $\langle \neg A\rangle$ a possible truthmaker y, then $\langle A \vee \neg A\rangle$ contains both x and y and so, by the sum closure condition, also contain $x \sqcup y$. But $x \sqcup y$ is a truthmaker for $\langle A \vee \neg A\rangle$! This is an entity that cannot possibly exist. It is a sum of incompatible entities, from different possible worlds. If we want to avoid commitment to such entities, we should restrict the sum closure principle to possible entities: if $x, y \in \langle A\rangle$ and $x \sqcup y$ possibly exists, then $x \sqcup y \in \langle A\rangle$. (In §4, however, I will offer a reason for wanting impossible entities in the theory.) We may also want to ensure that single propositions are *convex*: if $x, z \in \langle A\rangle$ and some part y of z has x as a part (that is, $x \sqsubseteq y \sqsubseteq z$) then $y \in \langle A\rangle$. (Fine (2014b) discusses convexity in relation to content; Fine and Jago (2017) discuss convexity in the context of truthmaker semantics.)

If a set satisfies these conditions, then it counts as a single proposition. That allows many, many arbitrary sets to count as propositions. Take the closure of an arbitrary set, $\{x_1, \ldots, x_n\}$, under the conditions just listed. This is the proposition that x_1, or ..., or x_n exists. If the set is upwards-closed under mereological summation, this disjunction can be made true in virtue of any of its disjuncts, or any combination of them, being made true. But there may be additional ways to characterise this set. In general, if (the closure of) $x_1, \ldots, x_n$ are all the possible truthmakers for A, then (the closure of) that set is the proposition *that A*. So each proposition is identified with the proposition asserting that at least one of its truthmakers exists, and hence with the proposition that it is made true.

These conditions apply to single propositions, and they apply equally to each component, $|A|^+$ and $|A|^-$, of a double proposition. In addition, we had better rule out any possible entity being in both sets of a double proposition. If some possible x were a member of both $|A|^+$ and $|A|^-$, then it would be possible for both $\langle A \rangle$ and $\langle \neg A \rangle$ to be true simultaneously. But this is not possible, so no possible entity can be in the overlap of $|A|^+$ and $|A|^-$. ($|A|^+$ and $|A|^-$ may overlap only if we accept impossible entities.)

4. Metaphysical Worries

There is a serious metaphysical worry facing double propositions. We might identify the double proposition $\langle A \rangle$ with an ordered pair, $(|A|^+, |A|^-)$. But we might instead identify it with $(|A|^-, |A|^+)$. Which identification is correct? For the purposes of semantics, either approach is fine. But my interest here is predominantly in the metaphysics of propositions. If we want to know what propositions are, metaphysically speaking, then one choice is right, one wrong; but there is no way to say which.

(If we further identify ordered pairs with sets, we face an additional issue. In general, we can code the pair (x, y) as $\{\{x\}, \{x, y\}\}$, or as $\{\{b\}, \{a, b\}\}$, or as $\{a, \{a, b\}\}$, $\{b, \{a, b\}\}$, or as $\{\{0, a\}, \{1, b\}\}$. Why think that one way gets the nature of propositions right, rather than the others?)

If you do not see a problem here, try this. Consider the pair, ($\{$*that Bertie is snuffling*$\}$, $\{$*that Bertie is not snuffling*$\}$). Assume (for the moment) that this is a proposition. Is it *that Bertie is snuffling* or *that Bertie is not snuffling*? Why? There cannot be any intrinsic differences in the composition of those sets to mark the difference, for the negation of a proposition $\langle A \rangle$ consists in those very same sets, $|A|^+$ and $|A|^-$, but with the order switched: $|\neg A|^+ = |A|^-$ and $|\neg A|^- = |A|^+$. So it seems we need to stipulate which set in the pair comes first, the truthmakers or the falsemakers. Yet there is nothing in the nature of propositions, or the nature of *truth*, which dictates any priority between truth and falsity. The problem is insoluble.

If we cannot make metaphysical sense of double propositions, then we will have to make do with single propositions. But then we must face again the issue of distinct but necessarily false propositions, raised in §3. How should we distinguish them, given that they have no possible truthmakers? We must drop the restriction to possible entities, by allowing propositions to include states of affairs which could not possibly obtain.

Above, we met one way to have impossible entities in our ontology. If possible states of affairs *that A* and *that ¬A* exist, then so does their mereological sum. I take sums of states of affairs to be conjunctive states of affairs: in this case, the (necessarily non-obtaining) state of affairs *that A ∧ A*. This state of affairs *that A ∧ A* is distinct from *that B ∧ B* whenever *that A* and *that B* are distinct states of affairs. And that in turn is enough to distinguish $\langle A \wedge A \rangle$ from $\langle B \wedge B \rangle$.

Other impossible cases are not explained so easily. Take the necessarily false proposition $\langle 1 = 2 \rangle$. One might think that the very identities of those numbers, 1 and 2, is what makes this false. But on the single-proposition approach, we are limited to possible and impossible truthmakers. What would an impossible truthmaker for $\langle 1 = 2 \rangle$ look like?

One suggestion is this: the (necessarily non-obtaining) state of affairs *that {1, 2} is a singleton.* For, were {1, 2} a singleton, 1 would be identical to 2. But this suggestion gets things the wrong way around. The identities of its members make a set the set it is. It is not the properties of the set that fix the identities of its members. Another approach is to take the impossible truthmakers for ⟨1 = 2⟩ to be a state of affairs universally quantifying over properties: *that, for any F, F1 iff F2.* But again, this gets the explanation the wrong way around. It is not that *a = b* because *a* and *b* share all their properties; rather, any property of *a* is a property of *b* because *a = b*.

A better approach is available for those who take mathematical entities to be identical to points in a structure. Then, the identity of 1 and 2 is given by relational, structural facts. The (necessarily non-obtaining) state of affairs *that 1 = 2* would be a conjunction of structural facts, identifying the 1-role with the 2-role. Just how this is done (and whether it is plausible) will depend on the details of one's particular structuralist theory.

If there are necessarily existing primitive entities, whose identities are not metaphysically analysable or grounded in more basic facts, then this kind of approach will not cover all cases. We will then be forced to admit some strange ontological ideas. Perhaps there is an *identity* relation, so that *that 1=2* involves the (impossible) instantiation of *identity* with 1 and 2. That is an ugly solution, since for most everyday metaphysical purposes, no *identity* relation is required. Facts of identity are given by the identical entities themselves (which is to say, by each and every thing).

The double propositions account has a much more elegant solution to offer. It treats ⟨1 = 2⟩ as the empty set (since nothing could make 1 = 2) paired with {1 ⊔ 2} (since 1 and 2 together make it the case that 1 ≠ 2). So each account on offer—double propositions, or single propositions with impossible states of affairs—has its benefits and its drawbacks. The former requires us to stipulate, in what would seem an *ad hoc* way, which set in each pair is to count as the truthmakers, which the falsemakers. The latter will probably require the introduction of some dubious ontology. Such is the way in metaphysics. I will put my money on single propositions (with impossible states of affairs).

Both approaches face a further difficulty with propositions such as:

(3) ⟨Propositions exist⟩
(4) ⟨Sets exist⟩.

One might expect the truthmakers for (3) to be all propositions, and truthmakers for (4) to be all sets. Indeed, that result falls out of a general principle: existential truths are made true by the truthmakers for their instances. But this is incompatible with (3) and (4) themselves being sets. Since (3) is a proposition, it would contain itself, contrary to the axiom of regularity (which rules out circular membership chains, $x \in \ldots \in x$. Similarly, if (4) is a set, then it would contain itself; but it cannot.

One may respond that some versions of set theory—non-well-founded theories—allow sets to contain themselves as members (Aczel 1988). I am not tempted by that route. For one thing, I am not sure we can make metaphysical sense of non-well-founded sets, given that sets are grounded by their members. For another, our theory of propositions should not dictate what fundamental mathematical theories should look like.

Even if we set these worries to one side, (4) is simply too big to be a set. If it contained all its truthmakers, it would be the set of all sets. But on pain of contradiction, there can be no such set. There is a similar worry for (3). For each entity x, there exists the proposition ⟨x exists⟩. The possible truthmakers for this are x itself (plus x's grounds). But (3) purports to contain all such sets, and hence purports to be at least as large as the set of all sets. There cannot be such a set.

We might respond with a theory that accepts proper classes, bigger than any set. But then we face the issue: how do we assert the existence of such classes? We are assuming that the proposition ⟨x exists⟩ is a set-or-class with x as a member. But proper classes are, by definition, members of no set-or-class. So if x is a proper class, then there is no proposition (qua set-or-class) asserting its existence.

These issues run deep. But they are a problem for everyone (who believes in sets). Even if you think there is no such thing as propositions, you still need to explain how the sentences

(5) There are sets
(6) All sets have Ø as a subset

get to be true. These truths require a domain of quantification, which contains all the entities quantified over by those truths. But, on the face of it, both sentences quantify over all sets. That would imply a domain of quantification—a set—containing all sets, which is impossible. (If you want to escape by taking the domain of quantification to be a proper class, just change 'set' to 'class' in the examples to re-introduce the problem.)

Somehow, we meaningfully talk about sets using 'all sets' without thereby including all sets in the domain of quantification. The domain of 'all sets' cannot include the domain of quantification itself. Similarly, the domain of 'some set' cannot include the domain of quantification itself. That seems to be a fact about how the quantifiers work. Their semantics allows 'all sets' and 'some set' to range over all sets except the set specifying that very range.

I propose that the same goes for the quantifiers in (3) and (4): they range over all sets, except the very sets specifying those ranges. Those range-specifying sets are precisely (3) and (4), respectively. So neither (3) nor (4) quantifies over itself, and hence neither is a truthmaker for itself. Both are genuine propositions, on this account. This avoids both the self-membership and the cardinality worry. And, importantly, the result is a consequence of the general semantics for the quantifier 'there are Fs'. This is a piece of the theoretical jigsaw put in place prior to the account of what propositions are. It does not require us to fiddle with our theory of propositions.

5. What are Merely Possible States of Affairs?

I have claimed that propositions are sets, or pairs of sets, of possible (and perhaps impossible) entities. Typically, these entities are states of affairs. On pain of contradiction, not all of the states of affairs thereby quantified over can obtain. But what on earth is a state of affairs that does not obtain? Here are three potential options.

OPTION 1: There exist merely possible concrete states of affairs, making up other possible worlds. 'Obtaining' (relative to world w) means ex-

isting at (as part of) world *w*. Non-obtaining states of affairs (relative to our world) are otherworldly states of affairs.

OPTION 2: Some states of affairs do not exist (but remain legitimate objects of quantification). The obtaining states of affairs are those that exist.

OPTION 3: There exist 'ersatz' states of affairs, in addition to the concrete ones. An ersatz state of affairs obtains when it corresponds to some concrete state of affairs.

These approaches are modelled on the main options in the metaphysics of possible worlds. The first takes its cue from the genuine modal realism of Lewis (1986), McDaniel (2004), and Yagisawa (2010). On this approach, all possible worlds are ontologically on a par with our own. The second is a broadly Meinongian approach, defended (in the case of worlds) by Priest (2005). The third approach is based on ersatz modal realism (Adams 1974; Stalnaker 1976a), on which possible worlds other than our own are actually existing ersatz representations.

The genuine realist view of possible states of affairs is a non-starter (even for those who accept entities beyond those that actually exist). I am not wearing a hat, but I could have been. Both states of affairs, *that I am wearing a hat* and *that I am not wearing a hat*, are possible. According to genuine realism about possible states of affairs, reality includes both states of affairs, *that I am wearing a hat* and *that I am not wearing a hat*. Reality is inconsistent! And since the existence of a state of affairs makes the corresponding proposition true, the contradictory propositions ⟨I am wearing a hat⟩ and ⟨I am not wearing a hat⟩ would both be true.

One may respond that those possible states of affairs are parts of distinct possible worlds. No possible world contains both of them (because, although they're each possible, they are not jointly possible). What is possible is whatever obtains at some possible world. So the contradiction, I am wearing a hat and not wearing a hat, is not possible. Consistency is restored.

This response is no solution. A genuine realist (either about possible worlds or about possible states of affairs) needs some standpoint from which she can assert her thesis. But there is no possible world which contains all those entities in which she believes. They are distributed across all the possible worlds. So, if we insist strictly that what is possible is whatever obtains at some possible world or other, then genuine realism (either about worlds or about states of affairs) is ruled impossible from the get-go.

Note that the general problem here does not depend on having negative states of affairs in the ontology. Suppose there is no (actual or possible) negative states of affairs at all. Nevertheless, I could be wearing a completely red hat, and I could be wearing a completely green hat. Those possible states of affairs are metaphysically incompatible. If both exist, as genuine realism entails, then reality is impossible. And we cannot be having that.

The second option mooted above is Meinongian in spirit. It allows that some entities do not exist. On this view, it makes sense to talk about and quantify over entities which lack existence. The suggestion is that merely possible states of affairs be placed in this category. To avoid the problems faced by the genuine realist, the Meinongian must allow that some states of affairs do not act as truthmakers. Rather, she will say, only the existing ones make anything true.

For if all states of affairs act as truthmakers, and there are contradictory (but non-existent) states of affairs, then there are true contradictions (simpliciter), and we are back to the problems from above. So the Meinongian must say that an entity is a truthmaker only if it exists.

But then, what makes it true that some states of affairs do not exist? The only candidate truthmakers for

(7) ⟨Some states of affairs do not exist⟩

are states of affairs that do not exist. But we have just debarred all such states of affairs from acting as truthmakers. So (7) has no truthmakers. It is false. This entails that all states of affairs exist, contrary to the Meinongian view. Meinongianism about possible states of affairs is a non-starter.

Ersatz states of affairs avoid these worries. They count as states of affairs just as rubber ducks count as ducks, which is to say, not at all. They themselves do not constitute something's being the case. They merely *represent* real states of affairs. So they do not make propositions true (other than propositions about the existence of ersatz states of affairs).

What is an ersatz state of affairs, metaphysically speaking? The simplest approach identifies the ersatz state of affairs *that Fa* with the ordered pair containing F and a themselves, in that order: (F, a). Such entities look very similar to Russellian structured propositions (King 1995; 1996; Salmon 1986; 2005; Soames 1987; 2008). (They are identical, if we interpret Russellian propositions as set-theoretic tuples.)

As a consequence, the ersatz-truthmaker and Russellian approaches (almost) agree on what *singular* propositions are. For the Russellian, the singular proposition *that a is F* is a structured entity (perhaps a tuple) containing F and a themselves, in that order. On the ersatz-truthmaker approach, it is the singleton whose sole member is the ersatz state of affairs *that a is F*: $\{(F, a)\}$. But they differ radically on logically complex propositions. On the Russellian approach, a conjunctive proposition ⟨$A \wedge B$⟩ contains both conjuncts and the semantic value of '$\wedge$'. On the truthmaker approach, by contrast, it is the set $\{x \sqcup y \mid x \in \langle A \rangle, y \in \langle B \rangle\}$ of summed truthmakers for each conjunct.

The Russellian and ersatz-truthmaker approach share a common problem. The proposition *that Fa*, by its very nature, represents that *Fa*. But a mere list, tuple, or other structure consisting of F and a does not, by its very nature, represent that *Fa*. We may interpret some such structure as doing that, as we do for the sentence 'a is F'. Any interpretation given by us is contingent. We could have interpreted the structure some other way, or not at all. So that F-and-a-involving structure could have represented some other situation, or none at all. The Russellian must say that her proposition *that Fa* might have been some other proposition, or no proposition at all. Similarly, the ersatz state of affairs *that Fa* might have been some other ersatz state of affairs, or none at all, and so its singleton might have been some other proposition, or none at all. But propositions are not like that. Each is *essentially* the proposition representing whatever it represents. So neither the Russellian nor the ersatz-truthmakers account will do.

An adequate solution to our problem should be 'ersatz', in the sense that the entities standing for merely possible states of affairs cannot be genuine states of affairs. But they cannot be 'mere' representations of states of affairs, for these will not maintain the essential link we require between a proposition and what it represents.

My suggestion is this. States of affairs have natures, or essences, just as entities belonging to other categories do, and these natures provide us with a way of talking meaningfully about states of affairs that do not obtain. Here is not the place to argue for the general claim that individuals and properties have natures. But suppose they do and suppose, with Mackie (2006), that such natures are each sufficient for being a given thing (so that, necessarily, x's nature is y's nature only if $x = y$). Suppose further, with Plantinga (1974) that those natures exist necessarily, so that Socrates' nature exists even if Socrates does not. Then, I claim, we have the means to make sense of merely possible states of affairs. These assumptions are substantial commitments to take on. But without them, I cannot see how to understand merely possible states of affairs.

How should we understand the natures of states of affairs, given these assumptions? That will depend largely on our preferred account of states of affairs. Here is one tentative suggestion, built on an Armstrong-style *fundamental tie* view (Armstrong 1997; 2004). On that view, states of affairs have constituents, tied together to form a unified whole. (What about negative states of affairs? I refer the reader to Barker and Jago 2012.) The identities of these states of affairs are given by the identities of their constituents. The nature of *that a is F* is to be the positive state of affairs involving a's possessing F. In general, the nature of a state of affairs involves the nature of its constituents. My suggestion is that these natures are unified, structured wholes, just as the corresponding states of affairs are. The nature of *that a is F*, on this approach, involves the natures of a and of F, bound together by the nature of the fundamental tie.

If the natures of a and F are necessary existents, then the nature of *that a is F* will be too. So the nature of *that a is F* will exist regardless of whether a is F (and indeed, regardless of whether a exists and whether anything is F). For these states-of-affairs-natures are not themselves states of affairs (just as the nature of a given person is not itself a person). So the nature of *that a is F* does not make it the case that a is F. That is why it is consistent for that nature to exist, even if a is not F. a's being F requires the concrete state of affairs *that a is F* to exist, which is typically a contingent matter. We can then understand 'non-obtaining state of affairs' as picking out a state-of-affairs-nature which corresponds to no actual state of affairs. Let us say that a state-of-affairs-nature is *realised* (at a world) when the corresponding state of affairs exists (at that world).

On this approach, (single) propositions are sets of states-of-affairs-natures. Since these natures actually exist, we have no trouble explaining how propositions actually exist. A (single) proposition is true (at a world) when one of its members is realised (at that world). Importantly, this approach maintains the essential link between a proposition and what it represents, via its would-be truthmakers. A proposition (as a set) is essentially linked to its members, and each of its members (as a state-of-affairs-nature) is essentially linked to a (possible or impossible) state of affairs.

6. Same-Saying

We utter declarative sentences to say things to one another. What we thereby communicate is not the utterance itself, since we can say the same thing in different ways. As Frege says:

> If someone wants to say the same today as he expressed yesterday using the word "today", he must replace this word with "yesterday". … The case is the same with words like "here" and "there" (Frege 1956: 296).

Similarly, two people can say the same thing about someone or something in different ways. If I am talking to Anna about her knitting, I will use 'your knitting', she will use 'my knitting', and others might use 'her knitting' or 'Anna's knitting' to say the same thing: *that Anna's knitting is great*.

In these examples, we can contrast what is said with the particular *way* in which it is said. To bring out the idea, suppose Anna and Bob are arguing, Anna insisting that the planet now visible is Hesperus, whereas Bob insists that it is Phosphorus. There is clearly a sense in which they are not really disagreeing at all, for they are both correctly identifying the planet they see. Someone in the know may interject, 'stop arguing, you are saying the same thing!'

Nevertheless, both parties are genuinely informed when they come to learn that the planet is correctly called both 'Hesperus' and 'Phosphorus'. What they lacked was *a posteriori* knowledge, not linguistic competence. This shows that the notion of *what is said* in an utterance does not align with the meaning of the utterance, or with the speaker's beliefs, or with common knowledge in the conversation.

Under what conditions do utterances of two sentences '*A*' and '*B*' say the same thing? (Alternatively, under what conditions do speakers of those utterances say the same thing as one another?) A particularly interesting instance of this question occurs when '*A*' and '*B*' are logically related in a certain way. The question then becomes: which logical operations preserve same-saying? We would like answers to the following kind of question:

(8) Does '$A \lor (B \land C)$' say the same as '$(A \lor B) \land (A \lor C)$'?
(9) Does '$A \land A$' say the same as '*A*'? How about '$A \lor A$'?
(10) Does '$\neg\neg A$' say the same as 'A'?
(11) Does '$\neg(A \land B)$' say the same as '$\neg A \lor \neg B$'?

Call this general form of question *the logical same-saying issue*. To my knowledge, the issue has not been discussed in the same-saying literature.

The simplest answer to the general same-saying question is this:

(SAMESAYING) *A* says the same as *B* iff $\langle A \rangle = \langle B \rangle$.

Whether that is plausible depends on one's account of propositions. If propositions were sets of possible worlds, it would not be plausible at all. Saying that 1 + 1 = 2 is clearly not the same as saying that properties exist, or that Bertie is either snuffling or not. But all are necessary truths, and hence captured by the same set of possible worlds. Neither would (SAMESAYING) be plausible if propositions were Russellian structured entities. For on that view, 'Bertie is snuffling and wheezing' expresses a distinct proposition from 'Bertie is wheezing and snuffling', and yet these are two ways to say the same thing about Bertie.

I am going to argue that (SAMESAYING) is correct, so long as we understand propositions as truthmaker conditions. This approach provides a plausible general answer to the same-saying question. I will also argue that a truthmaker-based approach provides the only adequate answer to the logical same-saying issue.

If propositions are truthmaker-conditions, then (SAMESAYING) gets the cases involving indexicals and co-referring names right. The possible truthmakers

for 'today is sunny' are defined by taking 'today' fixed in the context of utterance. If today is Thursday 8th September 2016, then the relevant states of affairs capture all the possible ways in which Thursday 8th September 2016 could be sunny. The same goes for 'yesterday was sunny', uttered on Friday 9th. Its possible truthmakers are the same. The two sentences express the same truthmaker condition, and so (SAMESAYING) predicts, correctly, that they say the same thing. The same goes for the 'Hesperus'/'Phosphorus' and the 'my'/'your'/'her' cases.

More interesting is what the truthmaker-condition account says about the logical same-saying issue. It seems clear that distinct but logically related sentences can be used to say the same thing, *in virtue of the logical relation between them*. Any utterance of 'it is warm and sunny' says the same thing as an utterance of 'it is sunny and warm' in the same context. In general, in the same context, utterances of '$A \wedge B$' and '$B \wedge A$' say the same thing. We cannot explain this feature in terms of the necessary equivalence of '$A \wedge B$' and '$B \wedge A$' (or their equivalence in classical logic), because there are equivalent sentences, utterances of which do not say the same thing in a given context. Consider a mathematical example:

(12) I can colour in any map with just three colours, so that no two adjacent areas have the same colour.

(13) I can take one lemon and one orange, and thereby end up with three more fruits than I started.

Both claims are mathematically impossible, and hence (classically) equivalent. Yet utterances of (12) and (13) do not say the same thing. Each speaker claims to be able to do *different* (and, unbeknownst to them, impossible) things. The same holds of logical examples:

(14) The Liar is both true and false.

(15) Claims about large cardinal numbers are neither true nor false.

Here, both statements are classically unsatisfiable (and so classically equivalent), yet they say very different things. Suppose that (14)'s speaker is a dialethist, such as Priest (1979; 1987), who diverges from classical logic in rejecting the explosion principle (that everything follows from a contradiction). And suppose (15)'s speaker is a mathematical intuitionist, such as Dummett (1978; 1993), who rejects excluded middle. It is absurd to think that, in stating their different philosophical positions, they say the same thing as one another. So it is not the case that, in uttering any two classically equivalent sentences, the speakers thereby say the same thing as one another.

A much better account of the logic of same-saying is given by *exact truthmaker equivalence* (Fine and Jago 2017). 'A' and 'B' are exactly equivalent when they share all their truthmakers in all truthmaker models. This account predicts that, for each of the following pairs (a/b), utterances of them (in a common context) say the same:

(16a) It is cold and wet.

(16b) It is wet and cold.

(17a) Cath or Dave will turn up, and Ed will turn up.

(17b) Either Cath and Ed will turn up, or else Dave and Ed will.

(18a) Either Cath does not like Dave or she does not like Ed.

(18b) Cath does not like both Dave and Ed.

These pairs are intuitively clear cases of same-saying. So exact truthmaker equivalence looks to be in good standing as an analysis of same-saying.

There are other notions of logical equivalence which treat these cases correctly. *First-degree entailment* (Anderson and Belnap 1963) verifies equivalences (16)-(18), whilst distinguishing between classically equivalent contents. Indeed, relevant logics in general are often seen as ways to preserve content from premises to conclusion in an entailment. (Brady (2006) develops a semantics for a (weak version of) relevant logic in terms of *content inclusion*, for example.) If that is right, then one might expect relevant equivalence to amount to sameness of content, which should in turn amount to same-saying.

But first-degree entailment (and relevant logics in general) do not provide a good account of either same-saying or sameness of content. First-degree entailment treats both $A \wedge (A \vee B)$ and $A \vee (A \wedge B)$ as being equivalent to A. But these equivalences do not preserve what is said. Just consider:

(19) Bertie is snuffling, and either he is snuffling or Lenny is sleeping.
(20) Either Bertie is snuffling, or he is snuffling and Lenny is sleeping.

Neither says (just) that Bertie is snuffling, so neither says the same as 'Bertie is snuffling'. So relevant equivalence is not a good criterion for same-saying.

This point is powerful, since just about every logic treats both $A \wedge (A \vee B)$ and $A \vee (A \wedge B)$ as being equivalent to A. The truthmaker semantics for exact entailment is one of the few systems that draws semantic distinctions between A, on the one hand, and $A \wedge (A \vee B)$ and $A \vee (A \wedge B)$ on the other. So we have a strong argument in favour of analysing same-saying in terms of exact equivalence. Moreover, given the view of propositions as truthmaker-conditions, $\langle A \rangle$ and $\langle B \rangle$ are exactly equivalent iff $\langle A \rangle = \langle B \rangle$. So '$A$' and '$B$' say the same thing (in a context) iff $\langle A \rangle = \langle B \rangle$, just as (SAMESAYING) says.

7. Aboutness and Subject Matter

The truthmaker approach to propositions and same-saying also allows for a neat characterisation of a sentence's or proposition's *subject matter*, or what it is about. I will assume we have a fairly good grip on 'being about the same thing'. 'Hesperus'-sentences and 'Phosphorus'-sentences are both about Venus (perhaps amongst other things). We might characterise 'Bertie is snuffling' as being about Bertie and *snuffling*, or we might characterise it as being about whether Bertie is snuffling. (I take these to be distinct but complementary ways of talking about *aboutness*.)

In general, 'A' and 'B can be about precisely the same things and yet not say the same as one another. 'Bertie is snuffling' and 'Bertie is not snuffling' are both about Bertie and *snuffling* (or about whether he is snuffling), yet each says the opposite of the other. Nevertheless, being about the same things is a necessary condition for same-saying:

(ABOUTNESS) A says the same as B only if A and B are about the same thing(s).

As our starting point, let us say that 'Bertie is snuffling' is about whether Bertie is snuffling. We can identify what a sentence or proposition is about with a set of states of affairs. We then define the objects and properties it is about—Bertie

and *snuffling*, in our example—as those that appear as constituents of any of those states of affairs.

There are a number of ways we can implement the first step. The simplest would be to identify the subject matter of a sentence with the proposition it expresses. But this will give us the strange result that A and $\neg A$ have different and indeed incompatible subject matters (since the possible truthmakers for A and $\neg A$ do not overlap). This is the wrong result: A and $\neg A$ are incompatible precisely because they say opposite things about the *same* subject matter.

We improve matters by taking the subject matter of A to be the set of all its possible truthmakers and falsemakers: $|A|^{+} \cup |A|^{-}$. (If we adopt the double proposition account from §3, then we obtain A's subject matter by 'flattening' $\langle A \rangle$ into a single set, $|A|^{+} \cup |A|^{-}$.) This approach gives the correct results for negation: A and $\neg A$ coincide on their subject-matter.

This approach still gives strange results, however. It allows that $A \wedge B$ and $A \vee B$ can have different subject matters. They differ in their truthmakers (and falsemakers) because conjunction pairwise sums together elements from and, whereas disjunction takes their union, $|A|^{+} \cup |A|^{-}$. But this gives incorrect results for subject matter: both are about whatever A is about, plus whatever B is about. They differ in what they say about that subject matter, but not in the subject matter itself.

There are two ways we can avoid this result. One is to take subject matter to be given by all the atomic parts of a sentence's truthmakers and falsemakers:

$$\{x \mid x \sqsubseteq \bigsqcup(|A|^{+} \cup |A|^{-}) \ \& \ x \text{ is atomic}\}.$$

(Here, $\bigsqcup X$ is the sum of all members of set X, and 'atomic' means 'having no proper parts'.) The other way is to take subject matter to be the sum of a sentence's truthmakers and falsemakers, $\bigsqcup(|A|^{+} \cup |A|^{-})$. Both approaches give similar results, given that subject matter (on the second definition) equates to the summed subject matter (on the first definition):

$$\bigsqcup\{x \mid x \sqsubseteq \bigsqcup(|A|^{+} \cup |A|^{-}) \ \& \ x \text{ is atomic}\} = \bigsqcup(|A|^{+} \cup |A|^{-}).$$

One benefit of the second approach is that it allows us to speak of *the* subject matter of a sentence: a unified entity. (On the first approach, by contrast, subject matter is a set, typically containing a plurality. It is somewhat strange, in general, to identify *the* subject matter of a sentence with a set.) The second approach also allows us to make sense of something's literally being a part (as opposed to a member) of a sentence's subject matter.

This approach to subject matter allows us to make sense of notions like *content inclusion* (Fine 2014a, 2014b; Yablo 2014). If we identify A's subject matter with, then A's subject matter includes B's just in case:

$$\bigsqcup(|B|^{+} \cup |B|^{-}) \sqsubseteq \bigsqcup(|A|^{+} \cup |A|^{-}).$$

This notion of inclusion, based on subject matter, ignores whether that subject matter is being affirmed or denied. So, for example, $A \wedge B$'s subject matter will include A's, even though the latter content is incompatible with the former. But we can also define a notion of content inclusion which avoids this consequence. We might say that A's content includes B's content just in case:

$$\bigsqcup |B|^{+} \sqsubseteq \bigsqcup |A|^{+} \text{ and } \bigsqcup |B|^{-} \sqsubseteq \bigsqcup |A|^{-}.$$

On either approach, the notion of content inclusion allows us to analyse locutions like, '*A* is partly about *B*' and '*B* is part of what *A* is about' in terms of *A* including *B*.

Content inclusion (on either approach) does not in general preserve exact truthmaking. *A* ∧ *B*'s content includes *A*'s content, yet an exact truthmaker for *A* ∧ *B* need not exactly truthmake *A*: it may have a *B*-relevant part, which is not relevant to *A*'s truth. (In other words, *A* ∧ *B* does not *exactly entail A* (Fine and Jago 2017).) Content inclusion does not even preserve truth. *A* ∨ *B*'s content includes *A*'s content, yet it may be that *A* ∨ *B* but not *A* is true.

Content inclusion can in turn be used to explain *partial truth*. The intuitive idea is that 'Hilary Putnam was one of the greatest female philosophers' is partly true (since he was one of the greatest philosophers), but not wholly true (since he was not female). A simple take on partial truth has it that *A* is (at least) partly true when it content-includes some (wholly) true *B*. 'Hilary Putnam was one of the greatest female philosophers' content-includes both 'Hilary Putnam was a philosopher' (true) and 'Hilary Putnam was female' (false) and so, on this analysis, is partly (but not wholly) true. (Fine 2016 gives an alternative account in terms of *analytic containment*, based on Angell 1989.)

There is clearly much more to be said about aboutness, subject matter, and the various notions of content inclusion. Yablo (2014) discusses these concepts in detail (he offers a fine-grained possible worlds-based account). My suggestion here is that the truthmaker approach offers a natural and elegant way to account for these concepts.

8. Conclusion

Propositions are not truth-conditions; they are truthmaker conditions. Metaphysically, truthmaker conditions are sets of the natures of actual and merely possible entities (typically, but not exclusively, states of affairs). Working with the natures of entities (rather than the entities themselves) allows us to capture the identities of merely possible entities without descending into paradox. Logically, the identity conditions on propositions is given by the logic of strict truthmaker equivalence. And semantically, the theory of propositions as truthmaker conditions illuminates *samesaying*, *subject matter*, and *aboutness*.

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