

# Kant on the Analyticity of Logic

*Costanza Larese*

*Scuola Normale Superiore, Pisa*

## *Abstract*

This paper calls into question the traditional interpretation that logic is, according to Kant, analytic. On the basis of a reconstruction of the salient features of both Kant's theory of analyticity and conception of pure general logic, it is shown that Kant does not apply the analytic-synthetic distinction to logical judgments at all. Moreover, applying Kant's definitions beyond his reasons for leaving the matter unsolved leads to the result that many logical judgments are neither analytic nor synthetic.

*Keywords:* Kant, Analytic-synthetic distinction, Pure general logic, Formality of logic, Containment criterion.

## 1. Introduction

In his *Wissenschaftslehre*, Bolzano writes: “Concerning logic, K.[ant] claimed that it (i.e., pure, general logic) consisted of nothing but analytic judgments” and adds “I cannot agree with this finding: rather, it seems to me that logic contains a considerable number of synthetic propositions” (Bolzano 2014, §315, vol. III: 161-62). The latter claim results from his criticism of Kant's notion of analysis together with his conception about the nature of logic. On the contrary, the idea that logic, according to Kant, is analytic is something that is taken for granted and Bolzano does not justify this assumption.

A similar position is taken by Frege. His work is devoted to the program of reducing arithmetic to logic, which amounts to showing, against Kant, that arithmetic is analytic. But Frege neither feels the need to specify that, according to his own definition of analyticity (Frege 1960, §3: 4), logical truths turn out to be analytic, nor to compare this outcome of his theory with Kant's position. That logic, according to Kant, is itself analytic is an unspoken assumption that works behind the scenes of the logicist program. This heritage is accepted by the Vienna Circle. Again, the critical target is Kant's synthetic *a priori*: this category, which has already been impoverished by Frege's thesis that arithmetic is analytic, is now rejected in toto (Carnap, Hahn and Neurath 1973: 308). But, once more, neither the idea that logic is analytic, nor the alleged Kantian origin of this thesis is called into question.

What is perhaps more surprising is that this belief is not shaken even in a work like Hintikka's *Logic, Language-Games and Information* that aims at vindicating Kant's position against the attacks of the Vienna Circle. Here (1973: 182), Hintikka argues that some polyadic first-order inferences are synthetic *a priori* and that Kant would have considered these modes of reasoning mathematical, rather than logical. This is indeed a vindication of Kant's claim that mathematics is synthetic *a priori*. However, the flip side of Hintikka's reasoning is that analytic first-order arguments correspond with arguments that Kant treated in logic. Again, Hintikka does not justify his claim that Kant considered the logic of his days to be analytic.

Despite rare notable exceptions,<sup>1</sup> this interpretative attitude has survived until now. For example, Hanna (2001: 140) states that "Kant also holds that all the truths of logic—that is, all the truths of what he regarded as logic—are analytically true" and Anderson (2015: 103) states Kant's alleged suggestion that formal general logic is analytic.

The aim of this paper is to shed light on this thorny issue: is it really the case that logic is, according to Kant, analytic? In order for this question to make sense, it is necessary to specify, first of all, at least some defining features of Kant's theory of analyticity.

## 2. The Containment Criterion of Analyticity

In a famous passage of the Introduction to the first *Critique*, Kant presents his analytic-synthetic distinction in the following terms:

In all judgments in which the relation of a subject to the predicate is thought (if I consider only affirmative judgments, since the application to negative ones is easy) this relation is possible in two different ways. Either the predicate *B* belongs to the subject *A* as something that is (covertly) contained in this concept *A*; or *B* lies entirely outside the concept *A*, though to be sure it stands in connection with it. In the first case I call the judgment **analytic**, in the second **synthetic**.<sup>2</sup>

This excerpt makes clear that the containment criterion of analyticity cannot apply to judgments whatsoever. First, it is restricted to true judgments. Analyticity in terms of containment is a sufficient reason for the truth of judgments: as a result, false judgments cannot be analytic. Second, it applies only to affirmative judgments. Nevertheless, the definition can be easily extended as to contemplate also negative judgments, which might be said to be analytic if the predicate is incompatible with the concept of the subject. Third, it is restricted only to categorical judgments, namely judgments of the subject-predicate form.

Kant's theory of analyticity has been attacked mainly on the ground of the third restriction. The containment criterion soon appeared too narrow. For example, Frege finds in this restriction one of the reasons for what he took to be Kant's misunderstanding of the status of arithmetical judgments: "Kant obviously—as a result, no doubt, of defining them too narrowly—underestimated the value of analytic judgments" (1960, §88: 99-100). Many denied the very fact that Kant intended

<sup>1</sup> See De Jong 2010: 250, and Burge 2005: 388.

<sup>2</sup> (CPR) A6-7/B10. All quotations from Kant's *Critique of Pure Reason* follow the English translation in Kant 1998 and are cited by page numbers in the original first (A) and second (B) editions preceded by the acronym (CPR).

his analytic-synthetic distinction to apply only to categorical judgments.<sup>3</sup> On the one hand, some maintained that the containment criterion was nothing but a proper part of Kant's theory of analyticity, which would be extended thanks to more comprehensive criteria.<sup>4</sup> On the other hand, some tried to balance the weight of the textual evidence given by the very beginning of the quotation above with other Kantian *loci*, that have been read as proofs of Kant's supposed intention to apply his distinction to judgements of any kind.<sup>5</sup>

Two are the main texts that are usually interpreted as saying that Kant intended to apply his analytic-synthetic distinction to any kind of judgments:

Judgments may have any origin whatsoever, or be constituted in whatever manner according to their logical form, and yet there is nonetheless a distinction between them according to their content, by dint of which they are either merely *explicative* and add nothing to the content of the cognition, or *ampliative* and augment the given cognition; the first may be called *analytic* judgments, the second *synthetic* (Kant 1997: 16).

Every existential proposition is synthetic ((CPR) A598/B626).

Proops (2005: 592ff.) has persuasively shown that the two passages can be given a different reading. The former does not mean that the analytic-synthetic distinction applies to judgments regardless of their logical form, but rather that any subject-predicate judgment may have any *degree of distinctness* whatsoever and still be appropriately classified as analytic or synthetic. The latter, together with Kant's famous claim that existence is not a predicate, seems to suggest that there are non-categorical judgments that are synthetic. Nevertheless, Proops points out that Kant, in his criticism of the ontological argument, holds that existence is not a "real predicate, i.e., a concept of something that could add to the concept of a thing" ((CPR) A599/B627), but he does not say that existence is not a *logical* predicate. On the contrary, Kant claims that "anything one likes can serve as a logical predicate" ((CPR) A598/B626): existence included.

Therefore, textual evidence is not overwhelming. But conceptual motivations are decisive. Kant's theory of analyticity is restricted to categorical judgments simply because it is not possible to apply the containment criterion to judgments that are not of the subject-predicate form. Only categorical judgments require the relation of thought between concepts, between a subject and a predicate; on the contrary, disjunctive and hypothetical judgments consider the relation of thought of judgment to judgment(s).

Moreover, if Kant had intended to apply his distinction via containment to all kinds of judgments, he could have worked out a strategy to reduce non-categorical judgments to categorical ones along the Leibnizian lines. But Kant did not go down that road. Hypothetical and disjunctive judgments are enumerated, together with categorical ones, under the heading "relation" in Kant's table of judgments and Kant insists that all the twelve forms of judgments must be recognized as primitive.<sup>6</sup>

<sup>3</sup> See e.g. Kneale and Kneale 1962: 357.

<sup>4</sup> See e.g. Hanna 2001: 145.

<sup>5</sup> See e.g. Anderson 2015: 20.

<sup>6</sup> See Kant 1992, §105: 601.

Since it applies only to true, (affirmative), categorical judgments, Kant's analytic-synthetic distinction via the containment criterion is not exhaustive and, as a consequence, there are some judgments that are neither analytic nor synthetic. This characteristic of Kant's classification is surely a disappointment for most of the twentieth-century philosophers, but probably not for his contemporaries. The main critical target of the *Critique* is the metaphysics based on the Leibnizian predicate-in-subject theory<sup>7</sup> and, given the close relationship between containment and categorical judgments, it is sufficient for Kant to focus on judgments of the subject-predicate form. In other words, Kant's "chief concern is to argue for the syntheticity of certain judgments", such as the claims of mathematics, natural sciences and metaphysics, "that in his days would have been assumed to have subject-predicate form" (Proops 2005: 589).

Beside the charge of narrowness, Kant's theory of analyticity has been accused for a long time of both psychologism and obscurity. While the former criticism can be easily dismissed,<sup>8</sup> the latter is more serious. It plays an important role in Bolzano's analysis of the Kantian definition<sup>9</sup> and has become a cliché after Quine's attack in his influential *Two Dogmas of Empiricism*, where he states that Kant's formulation "appeals to a notion of containment that is left at a metaphorical level" (Quine 1951: 21). Only recently some scholars<sup>10</sup> have challenged this interpretative trend by pointing out that the containment criterion, far from being a metaphorical formulation, is instead a precise notion. As Anderson explains, Kant follows the Wolffian tradition and clarifies the standard notion of containment by appealing to the theory of logical division of concepts and Porphyrian concept hierarchies.

According to the traditional theory of concepts, each genus is said to be "contained in" its species and each species is "contained under" its genus. For Kant, containment relations are thus ordered in a hierarchy of genera and species, where each genus is contained in its species and each species is contained under its genus. While admitting a *summum genus*, Kant denies the possibility of lowest concepts,<sup>11</sup> because, since concepts are general, their extension "must at every time contain other concepts, i.e., subspecies, under itself" ((CPR) A656/B684). By virtue of this relation between containment and the theory of genus and species, the rules of logical division can be applied to the standard notion of containment.<sup>12</sup>

<sup>7</sup> See Anderson 2015.

<sup>8</sup> On this point see e.g. Hanna 2001: 155ff.

<sup>9</sup> See Bolzano 2014, §148, vol. II: 61-2.

<sup>10</sup> In particular, Anderson 2015: Part I, and De Jong 1995.

<sup>11</sup> Kant believes that there can be no singular concept. The clash with his table of judgments, which distinguishes between universal, particular and *singular* judgments according to the quantity of the subject concept, is only apparent. Kant argues that "It is a mere tautology to speak of universal or common concepts—a mistake that is grounded in an incorrect division of concepts into *universal*, *particular*, and *singular*. Concepts themselves cannot be so divided, but only *their use*" (Kant 1992, §1: 589). Thus, every concept is general, but might be *used* to think about singular things.

<sup>12</sup> This theory accounts also for the fact that Kant, in the *Groundwork of the Metaphysics of Morals* (Kant 1902, IV 417), takes hypothetical imperatives to be analytic. The point is that the analysis of the concept of "willing the end" contains the concept "ought to will the necessary means" (notice that this formulation preserves the possibility of willing an end and, at the same time, not willing the necessary means, which is indispensable for an imperative to be an imperative).

The divisions, which are based on the Aristotelian definitions, are governed by the rule that the species exhaust the divided genus and exclude one another: divisions are exhaustive and exclusive disjunctions. Therefore, the relation of two concepts is either of complete inclusion or of total exclusion: partial overlaps are not admitted in these concepts' hierarchies. As a result, judgments that connect any two concepts will be either true, in the case of total inclusion, or false, in the case of total exclusion. Containment is not a metaphor, but rather a technical criterion deeply rooted in theories that were widely available in Kant's days.

### 3. Clarification, Identity and Contradiction

In the first *Critique*, Kant's distinction between analytic and synthetic judgments is defined not only in terms of containment, but also according to three more criteria, namely, clarification, identity and contradiction:

1. The clarification criterion is characterized in the Introduction to the *Critique* combining both a negative and a positive requirement. The former is that analytic judgments "through the predicate [...] do not add anything to the concept of the subject"; the latter is that analytic judgments break the concept of the subject up "by means of analysis into its component concepts, which were already thought in it (though confusedly)" ((CPR) A7/B11; see also Kant 1997: 19).
2. Again, in the Introduction, Kant explains that in analytic judgments (affirmative ones) "the connection of the predicate is thought through identity" ((CPR) A7/B10-11).
3. In the second chapter of the *Analytic of Principles*, Kant seems to suggest that analytic judgments can be known through the only means of the principle of non-contradiction ((CPR) A151-2/B190-1).

But what is the relationship between Kant's four criteria of analyticity?<sup>13</sup>

The clarification criterion can be reduced to the containment definition, which constitutes its fundamental idea. The deep link between the two versions of analyticity can be mostly appreciated considering the positive feature of the definition above: the clarification of the concepts' intensions involved in a certain analytic judgment, which is obtained through conceptual analysis, consists of showing that the predicate concept is contained in that of the subject. Despite of the immediacy of this argument, some scholars have objected that clarification and containment do not have the same extension, because the former would be characterized by an epistemic flavour that the latter would lack.<sup>14</sup>

However, this criticism can be easily dismissed. It cannot be denied that, according to the clarification criterion, analytic judgments are not cognitively empty in so far as the process of analysis explicates the concepts involved by making distinct their conceptual marks. But this feature of analytic judgments emerges

<sup>13</sup> This is one of the major topics in the literature. Since Kant's four criteria of analyticity do not seem to be equivalent, scholars have discussed on whether the set of Kant's formulation is consistent after all. Some, such as Hanna 2001: 124, argued that each definition "merely brings out a different aspect of a single, internally consistent, defensible Kantian theory". Others believed that Kant's criteria cannot be reconciled and identified one of them as the conceptually fundamental or most mature formulation. See e.g. Anderson 2015: 16, Proops 2005, Allison 2004: 89ff.

<sup>14</sup> See Proops 2005: 602, and Allison 2004: 90.

from the containment criterion as well, where the predicate-concept is explicitly said to be “covertly” ((CPR) A6/B10) contained in the subject concept. The distinction via clarification is still a distinction between two kinds of propositional content, as it is for the containment criterion, and not of two kinds of cognitive procedures. Clarification is a characterization in epistemic terms of the same logical distinction based on the containment criterion.

The relation between containment and the identity criterion is problematic when identical judgements, such as “man is man”, are taken into account. On the one hand, according to containment (and clarification), analytic judgments are endowed with cognitive content and are not trivial or tautologous. In particular, the predicate concept must at least be different from the subject concept, for otherwise there is no room for any kind of clarification whatsoever. On the other hand, it is obvious that the identity criterion classifies identical judgments as analytic. Kant himself oscillates on this point.<sup>15</sup> While both in the *Critique* and in the *Prolegomena* he clearly holds that “a = a” is analytic,<sup>16</sup> in other *loci* of his work he rejects the thesis of the analyticity of identical judgements.<sup>17</sup>

Identical judgments notwithstanding, containment and identity are strictly connected. In partial identities, such as “all bodies are extended”, the predicate concept is partially identical with the subject concept, because the relation of full identity subsists only between the conceptual notes of the predicate concept and a proper part of the conceptual marks of the subject. But this is just a different way of phrasing the containment criterion, because the predicate concept, being a part of the subject concept, is contained in it. Therefore, containment is the fundamental idea at the basis of the identity criterion, although the latter excludes any consideration of epistemic nature and classifies identical judgments as analytic.

The contradiction criterion has been frequently identified as the best among Kant’s versions of analyticity.<sup>18</sup> Two are the main reasons that explain its happy fortune. First, it seems more inclusive than the containment criterion, because it is not restricted to categorical judgments. Second, it is closer to contemporary appeals to the class of logical truths in providing a definition of analyticity. Despite this long interpretative tradition, some scholars have shown that the principle of contradiction is not a definition of analyticity at all, but rather an instrument for knowing the truth of analytic judgments.<sup>19</sup> Textual evidence is here determining:

Now the proposition that no predicate pertains to a thing that contradicts it is called the principle of contradiction, and is a general though merely negative *criterion of all truth* [...]

But one can also make a positive use of it, i.e., not merely to ban falsehood and error (insofar as it rests on contradiction), but also *to cognize truth*. For, **if the judgment is analytic**, whether it be negative or affirmative, *its truth must always be able to be cognized* sufficiently in accordance with the principle of contradiction. For the contrary of that which as a concept already lies and is thought *in the cognition of the*

<sup>15</sup> This fact has been explained in different ways. For example, De Jong (1995: 629-30) holds that, strictly speaking, tautological judgments for Kant are neither analytic nor synthetic. Proops (2005) proposes instead a diachronic reading of Kant’s position.

<sup>16</sup> (CPR) B17 and Kant 1997: 19. See also Kant 1992, §37: 607.

<sup>17</sup> See e.g. Kant 1902, XX 322.

<sup>18</sup> See e.g. Kneale and Kneale, 1962: 357-58.

<sup>19</sup> See e.g. De Jong 1995 and Proops 2005: 603.

*object* is always correctly denied, while the concept itself must necessarily be affirmed of it, since its opposite would contradict the object.

Hence we must also allow the **principle of contradiction** to count as the universal and completely sufficient **principle of all analytic cognition**; but its authority and usefulness does not extend beyond this, as a sufficient *criterion of truth*. For that *no cognition* can be opposed to it without annihilating itself certainly makes this principle into a *conditio sine qua non*, but not into a determining ground of *the truth of our cognition* ((CPR) A151-2/B190-1, emphasis added).

Kant's explanation of the role of the principle of non-contradiction does not find its place in the Introduction, together with containment, clarification and identity criteria, but only later on in the Analytic of Principles. In this passage, Kant is listing the uses of the principle of non-contradiction and he maintains that it is both a "negative criterion of all truth", meaning that it is a necessary condition for the truth of any judgment, and the "principle of all analytic cognition", namely the necessary and sufficient condition for the cognoscibility of analytic judgments. As the emphasised phrases make clear, Kant is careful in stressing its instrumental role as a criterion for establishing the truth of judgments and its epistemological function for determining the possibility of knowing analytic judgments.

Saying that an affirmative analytic judgment is known in accordance with the principle of contradiction does not mean that it is possible to derive an explicit contradiction from the negation of the judgment involved, but rather that the contradiction rests with the concept of the subject and the negation of the predicate. This is because the predicate is "already thought beforehand in the concept of the subject" (Kant 1997: 17), for if the predicate were not thought in that of the subject, then the denial of the former would not contradict the latter. This means that the ultimate reason for the epistemic function of the principle of contradiction in knowing the truth of analyticities is, once more, the relation of containment between the concepts involved in analytic judgments.

To sum up, the containment criterion, which applies only to true, (affirmative) and categorical judgements, is the central notion of Kant's theory of analyticity, not only because it is announced first and has an expositional priority over the other formulations, but also because the remaining criteria are founded on it and might be (completely or partially) reduced to it.

#### 4. Kant's Conception of Logic

The previous sections have specified the main features of Kant's theory of analyticity. But, in order to understand whether logic is really analytic for the author of the *Critique*, it is obviously necessary to delve into another preliminary issue, namely, Kant's conception of logic. In particular, two questions need to find answers. First, what counts as "logical" for Kant and what kind of logical notions did he possess? Second, how is logic conceived and which are the defining features of this discipline according to Kant?

In Kant's writings, the term "logic" refers to a variety of disciplines. In the introduction to the Transcendental Logic of the *Critique*, he first distinguishes between general and special logics. While the former "contains the absolutely necessary rules of thinking, without which no use of the understanding takes place, [...] without regard to the difference of the objects to which it may be directed", the latter "contains the rules for correctly thinking about a certain kind of objects"

((CPR) A52/B76). Then, he states that general logic might be pure or applied. The former abstracts “from all empirical conditions under which our understanding is exercised”; the latter undergoes “the subjective empirical conditions that psychology teaches us” ((CPR) A53/B77). At last, Kant introduces the discipline of transcendental logic that, unlike general logic, investigates the origin and the objective validity of the cognition of pure understanding and pure reason, through which we think objects completely *a priori*.

The relationship between pure general logic, which is the discipline that gets closer to both the traditional and the modern conception of logic and might thus be called logic in the strict sense of the term, and transcendental logic, which is Kant’s radical innovation and is a metaphysical discipline, is a debated issue.<sup>20</sup> Although there are no doubts that Kant attached the greatest importance to the latter, which covers the largest part of his first *Critique*, it is also fair to recall that the former is a constant presence in Kant’s intellectual life. Not only did he take several courses in logic as a student and deepen his logical knowledge while preparing his *venia legendi*, but he also wrote of logical issues and held numerous courses in logic during his forty-years teaching in Königsberg.<sup>21</sup> Nevertheless, a long interpretational tradition has claimed that Kant’s knowledge of logic was quite elementary<sup>22</sup> and his esteem for the latest developments of the discipline rather low.<sup>23</sup> A confirmation of this judgment might come from a closer look to what Kant thought belonged to the domain of the logical.

However, identifying in a precise way which topics were proper of pure general logic according to Kant is no easy feat. First, beyond *The False Subtlety of the Four Syllogistic Figures* (1762), the other logical work published during Kant’s time and associated with his name, that is to say, the so-called *Jäsche Logic* (1800), must be treated with caution and cannot be taken as a reliable statement of Kant’s view.<sup>24</sup> Something similar happens also for the other texts stemming from Kant’s logic lectures as well as for his handwritten *Reflexionen* on Meier’s handbook. Second, these texts suggest that the content of his lectures included matters that, on his own account, do not belong to logic proper. But it might be safely assumed that Kant accepted the traditional division of logic into three branches: the theory of concepts, of judgments and of inferences.

The theory of concepts provides a complete characterization of the basic unit of thought, through the discussion of crucial distinctions, such as matter and form, empirical and pure, *a priori* and *a posteriori*. Moreover, it includes all the tools needed to formulate the containment criterion, such as the definitions of “content” and “extension” of concepts, the connections between higher and lower concepts on the one hand and genera and species on the other. The theory of judgments concerns instead the relations between concepts. It focuses on the logical forms of judgments, classifying them accordingly to their quantity, quality, relation and modality, and discusses peculiar kinds of judgments. The theory of inferences consists of a restricted version of the Aristotelian syllogistic with a simple theory of disjunctive and

<sup>20</sup> See e.g. Tolley 2012.

<sup>21</sup> See e.g. Capozzi 2002: 59-113.

<sup>22</sup> See e.g. Bocheński 1961: 6, Kneale and Kneale 1962: 354, Young 1992: xvi, Hazen 1999: 92 and Lapointe 2012: 11.

<sup>23</sup> This at least seems to be suggested by Kant’s words at (CPR) B viii.

<sup>24</sup> On this point see Young 1992.

hypothetical inferences added on, such as *modus ponens* and *modus tollens*, together with a distinctive treatment of inductive inferences.

But Kant's original contribution to logic must be searched in his conception and philosophy of this discipline.<sup>25</sup> Pure general logic is for Kant a science in the strict sense of the term, namely "an exhaustive and a priori proven system of the merely formal rules of thought".<sup>26</sup> It is characterized by the following qualifying features. First of all, it is pure, because it disregards the empirical circumstances under which the understanding is applied and concerns only *a priori* principles. Second, it is general. This means that the rules of logic are necessary, because they have to be applied no matter what are the objects we are thinking about. On the one hand, they are constitutive for the understanding, in the sense that we cannot think at all without them;<sup>27</sup> on the other hand, they are normative, in the weaker sense that they prescribe how we have to think correctly.<sup>28</sup>

Third, Kant claims, probably for the first time,<sup>29</sup> that logic is formal, in the sense that, like grammar, it abstracts from the semantical content of thought. As a consequence, logic cannot yield any extension of knowledge about reality or objects (see (CPR) A60/B85). The presumption of employing general logic as a tool for extending knowledge, which is its dialectical use, "comes down to nothing but idle chatter, asserting or impeaching whatever one wants with some plausibility" ((CPR) A61/B86). Kant believed that formality was a direct consequence of the generality of logic<sup>30</sup> and many scholars have argued that the two notions, given some Kantian premises, ultimately collapse.<sup>31</sup>

Fourth, Kant holds that logic is a canon for thinking, which is "the sum total of the a priori principle of the correct use of [...] understanding and reason in general, but only as far as form is concerned" ((CPR) A796/B824). The thesis that logic is canon for thinking is a consequence of its three features mentioned above.<sup>32</sup> Since logic is pure, the rules for the correct use of understanding and reason are *a priori*. Since it is general, logic is constitutive and normative for thinking: as a result, it prescribes the correct use of understanding and reason and, in this sense, it is a "cathartic". Last, since it is formal, logic cannot be an organon, namely "a directive as to how certain cognition is to be brought about" (Kant 1992, §13: 528-29), but only a canon.

## 5. Kant on the Relationship between Pure General Logic and Analyticity

The relationship between analyticity and pure general logic consists of two distinct issues that have been frequently confused. First, the function that logic plays in Kant's definition and application of the analytic-synthetic distinction. Second, the question of whether logic itself is really analytic according to Kant.

<sup>25</sup> See Tiles 2004.

<sup>26</sup> See Lu-Adler 2018: 6.

<sup>27</sup> See Kant 1992, §12: 528.

<sup>28</sup> See Kant 1992, §14: 529.

<sup>29</sup> See MacFarlane 2002: 44-46.

<sup>30</sup> This can be easily seen in (CPR) A52/B76 and Kant 1992, §12: 585.

<sup>31</sup> See e.g. Lapointe 2012 and MacFarlane 2002: 32.

<sup>32</sup> Of course, for what has been said before, the premise that logic is formal is redundant in so far as it can be deduced from the fact that it is general.

Consider the first point. For what has been said above, it is clear that logic is the fundamental instrument that Kant employs for drawing the analytic-synthetic distinction. Containment, to which all of Kant's criteria of analyticity can be reduced (see Section 3), is a technical notion based on the theory of logical division of concepts (see Section 2). These were at the core of early modern logic: the theory of concepts is, at the same time, the foundation of the logical doctrine of elements and, in essence, a theory of concepts containment (see Section 4). Moreover, Kant's pure general logic is an instrument not only for defining the analytic-synthetic distinction, but also for applying it. The principle of non-contradiction, which is the logical principle *par excellence*, has the fundamental epistemological function of determining the truth, as well as the analyticity, of analytic judgments (see Section 2).

The role of logic as the fundamental instrument in Kant's thought for both defining and applying the notion of analyticity is probably one of the reasons why most interpreters, from the very beginning of Kant's critical reception until recent days, have concluded that Kant held that logic itself were analytic (see Section 1). Nevertheless, it is obvious that this conclusion is fallacious.

On the contrary, Kant nowhere seems to claim that logic is analytic. While he explicitly argues that judgments of experience, mathematics, natural science and metaphysics are synthetic (see (CPR) B11-8), Kant does not speak about the status of logic. The thesis that *Kant does not apply the analytic-synthetic distinction to logic* might be proven only in one way. Namely, it must be shown that the strongest passages usually taken to support the opposite view are not overwhelming. This is the case<sup>33</sup> for (CPR) A151-2/B190-1, which has already been quoted in Section 3, together with the following excerpts:

General logic analyzes the entire formal business of the understanding and reason into its elements, and presents these as principles of all logical assessment of our cognition. This part of logic can therefore be called an analytic ((CPR) A60/B84-5).

I understand by an analytic of concepts not their analysis, or the usual procedure of philosophical investigations, that of analyzing the content of concepts that present themselves and bringing them to distinctness, but rather the much less frequently attempted **analysis of the faculty of understanding** itself [...] for this is the proper business of a transcendental philosophy; the rest is the logical treatment of concepts in philosophy in general ((CPR) A65-6/B90-91).

General logic abstracts from all content of cognition, and expects that representations will be given to it from elsewhere, whenever this may be, in order for it to transform them into concepts analytically ((CPR) A76/B102).

As explained above, in (CPR) A151-2/B190-1, Kant affirms that the principle of non-contradiction is an instrument for knowing the truth of analytic judgments and nowhere states that logic is analytic. In (CPR) A60/B84-5, Kant suggests calling the formal part of general logic *an* analytic. His aim here is to distinguish the proper use of logic as a canon for judging from the use of it as an organon for the production of seemingly objective knowledge. He names the former part of logic an analytic to contrast it with the latter part of the discipline, that he calls "dialectic". In so doing, Kant himself explicitly states that he is following the

<sup>33</sup> See e.g. Hanna 2001: 140, and Anderson 2015: 103.

tradition. His choice of the term “analytic” is therefore meant only to underline the formal character of logic.

In (CPR) A65-6/B90-1, Kant suggests that the business of the logical treatment of philosophical concepts, unlike the transcendental one, is to analyse the content of concepts so as to clarify them. In (CPR) A76/B102, Kant is again distinguishing transcendental logic from pure general logic on the basis of the formality of the latter as opposed to the manifold of sensibility *a priori* that lies before the former. The point that Kant makes in the three latter passages is the same. In all of them he is simply underlining the formality of logic. But the formality of logic does not amount *per se* to say that the principles of logic are analytic (see Section 4). It only excludes that logic is synthetic (see Section 6).

Therefore, until proven otherwise, it can safely be concluded that in his writings Kant never claims that logic is analytic. A different question regards the theoretical reasons for his choice. Why doesn't Kant apply his analytic-synthetic distinction to logic? The answer to this question, as De Jong (2010: 250) suggests, must be searched in Kant's peculiar conception of logic proper. In particular, in its characterizing features of generality and formality. In so far as it is general, logic develops rules relative to form and, in so far as it is formal, logic abstracts from the content of thinking. As a result, logic cannot extend our knowledge of real objects. It shares with grammar<sup>34</sup> the destiny of being a propaedeutic, rather than a kind of knowledge:

Hence logic as a propaedeutic constitutes only the outer courtyard, as it were, to the sciences; and when it comes to information, a logic may indeed be presupposed in judging about the latter, but its acquisition must be sought in the sciences properly and objectively so called ((CPR) B ix).

Logic is a propaedeutic because it precedes any kind of knowledge: its rules have to be learnt and respected as a *conditio sine qua non* of any cognitive enterprise. In devising his analytic-synthetic distinction, Kant is primarily interested in doctrines, such as mathematics, metaphysics and sciences, that have some content of knowledge and his main purpose is to argue, against the Leibnizian predicate-in-subject theory, that judgments of those disciplines are synthetic. Determining the status of logic with respect to the analytic-synthetic distinction simply does not belong to Kant's *desiderata*.

## 6. Applying Kant's Definitions beyond Kant's Intentions

Although Kant, as a matter of fact, does not apply his analytic-synthetic distinction to logic, it is still possible to investigate whether logical judgments are analytic or synthetic *a priori* according to Kant's definitions and beyond Kant's reasons for leaving the matter unsolved. In other words, the following analysis is something that Kant did not want to pursue and did not consider a part of his philosophical strategy.

First of all, it can be shown that, according to Kant, logical judgments are not synthetic. This is simply because logical judgments cannot fit Kant's definition of synthetic judgments. In the latter kind of judgments, the concept of the predicate is not contained in the concept of the subject: rather, the former is “outside” ((CPR)

<sup>34</sup> See Mosser 2008.

A6-7/B10-1) or “beyond” ((CPR) A154-5/B193-4) the latter. Nevertheless, in order for grounding and justifying the truth of synthetic judgments, there must be some kind of connection between the two concepts involved, which must be different from the containment relation. This relation cannot be but indirect in that it has to link two concepts to one another by connecting them to a third and distinct element.<sup>35</sup> This third element is, for Kant, an object in which “the synthetic unity of their concepts could establish objective reality” ((CPR) A157/B196). But the appeal to an object for a logical judgment is what is explicitly excluded by the feature of formality that characterizes logic according to Kant (see Section 4).

Another, albeit partial, evidence comes from the following argument. Suppose, *ad absurdum*, that the principle of non-contradiction is synthetic. Since the principle of non-contradiction is the supreme principle of analytic judgments, it follows that analytic judgments can be derived from a synthetic principle. But now consider Kant’s thesis, put forward in the Introduction to the first *Critique* ((CPR) B14), that what can be proved from a synthetic judgment is itself synthetic. It turns out that analytic judgments are synthetic and that assuming the syntheticity of the principle of non-contradiction leads to the collapse of the analytic-synthetic distinction.

Notice that the claim that logical judgments are not synthetic is nothing more than a confirmation of the special status of logic in Kant’s epistemology. Logic, for Kant, is a science. But while all the other theoretical sciences, such as mathematics, are based on synthetic *a priori* judgments, this discipline, in so far as it is a body of necessary rules and a propaedeutic for thought, must be different. What is perhaps more surprising is that *at least some judgments of logic are not even analytic following Kant’s definition*. However, unlike for the question regarding the syntheticity of logical judgments, which can be given a compact (and negative) answer, the issue of the analyticity of logic requires to divide logical judgments into several categories and to consider them one by one.

The first class includes those principles (mainly belonging to the theory of concepts and the theory of judgments), which in Kant’s days belonged to logic with full right but have been excluded from the discipline during the development of modern symbolic logic. Examples of judgments of this class are “A concept is a universal representation” (Kant 1992, §1: 589) or “Propositions whose certainty rests on *identity* of concepts (of the predicate with the notion of the subject) are called *analytic* propositions” (Kant 1992, §36: 606).

Are this kind of judgments analytic? *Some* of them are analytic for sure. Consider the first of the examples above. In the passage of the first *Critique* commonly known as the “Stufenleiter” ((CPR) A320/B377), Kant divides the concept “representation” by following the traditional theory of the logical division of concepts (see Section 2), that is, by obtaining more specific concepts through the gradual addition of *differentiae specificae* to the higher ones. Through this method, a concept is said to be an objective representation with consciousness, whose relation to the object is mediate by means of a mark which can be common to several things. As a result of this investigation, the judgment “A concept is a universal representation” is analytic, because “representation” is the *genus* and “universal” is a *differentia* of the subject “concept”.

But is it possible to conclude that *every* judgment of this kind is analytic? Probably not. For sure none of them is synthetic. Nevertheless, some of them

<sup>35</sup> See e.g. (CPR) A9/B13 and A155/B194.

might still be non-analytic. First of all, because Kant's theory of analyticity is spelled out in terms of containment and containment is the basic notion of the theory of concepts. This means that asking whether the theory of concepts is analytic might raise problems of autoreferentiality, such as in the second of the examples above, since the theory of concepts seems to be a meta-theory, rather than an object theory. Second, in many cases it is not clear at all whether the way in which judgments are formulated is essential to their meaning. In the *Jäsche Logic*, many statements are obviously not categorical (and, as such, cannot be analytic at all), although they can be easily turned into the "S is P" form.

To conclude, it is probably necessary to consider judgments of the first class one by one to determine which of them are analytic and which are not. But with high probability the result of this procedure won't be worth the effort.

Consider now the second class of statements that, contrary to the former, includes judgments that are clearly part of modern symbolic logic, while being not "logical" according to Kant's notion of the term. All of those validities turning essentially on relations belong to this class. In Kant's time, categorical judgments were still considered to be the most fundamental judgments of logic, which was intrinsically monadic in character and not equipped with dedicated instruments for handling relations. As a result, truths turning essentially on relations are not analytic.<sup>36</sup> This is not because they are not logical, but rather because they cannot be properly reduced to categorical propositions. Nevertheless, it is still possible for them to be synthetic, in so far as they were excluded from the domain of the logical, which is, as argued above, the domain of the non-synthetic.

The third and last class of statements to examine is given by those that are logical both according to Kant's traditional conception of the discipline and for modern symbolic logic. This class includes not only propositional inferences, such as *modus ponendo tollens* and *modus tollendo ponens*, but also the hypothetical judgments that can be obtained by these inferences by considering the conjunction of their premises as the antecedent and the conclusion as the consequent. Both of them cannot be reduced to categorical judgements, because of their form: their soundness and validity rely on the relations between judgments independently of the concepts involved. As a result, the statements of this class are neither analytic nor synthetic or, equivalently, the analytic-synthetic distinction cannot apply to them.

One might think at this point that all but non-categorical and non-autoreferential judgments that for Kant belonged to logic are analytic according to his definitions. But this is not the case. For example, also categorical and identical truths would not count, according to Kant, as analytic, because they are not illuminating as the containment-clarification criterion prescribes. Thus, it seems that the class of the logical judgments that are neither analytic nor synthetic according to Kant's definitions of the terms might be wider than what was usually taken to be.

## 7. Conclusion

Against the prevailing view according to which Kant maintains that logic is analytic, this paper has shown that Kant does not apply his analytic-synthetic distinction to logic at all and that the grounds for this reticence about the status of logic

<sup>36</sup> This point has been variously acknowledged. See e.g. Anderson 2015: 99ff. This is also one of the main reasons that led scholars to reject the very fact that Kant's analytic-synthetic distinction is restricted to categorical judgments. See e.g. Hanna 2001: 145ff.

has to be searched in Kant's peculiar conception of the discipline. Even attempting an analysis that Kant did not think worth pursuing, it has been maintained that no logical judgment is synthetic *a priori* and that at least some logical judgments are not analytic. In other words, following Kant's definition of the analytic-synthetic distinction, it turns out that many logical judgments are neither analytic nor synthetic.

This result is not a mere question of terminology or classification. It bears relevant consequences, in the first place, on Kant's overall theoretical philosophy. To make one example, it calls into question the most common interpretation of Kant's invention of transcendental logic as an instrument devised in order to deal with his new category of the synthetic *a priori*, as opposed to the alleged analytic domain of pure general logic.<sup>37</sup> But it shows its importance also for adjusting the reading of an essential turning point in the history of logic. The idea that analytic judgments are either logical truths or can be turned into logical truths by replacing synonyms for synonyms, together with its consequence that logic is the example *par excellence* of an analytic discipline,<sup>38</sup> belongs to a much later view on the discipline of logic. The bond between logic and analyticity was not so tight at the beginning of the story.<sup>39</sup>

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<sup>37</sup> See the reconstruction in Tolley 2012: 419ff.

<sup>38</sup> See Carnap, Hahn and Neurath 1973.

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