

# Conditionals and Probability: Introduction

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This Special Issue aims to take stock of the philosophical debate about the relationship between conditionals and probabilities.

Three of the contributions (Nulvesu, Baratgin, Mura) revolve around de Finetti's truth-conditional theory of tri-events. The paper by Gilio (who was a student of his) and Sanfilippo concentrates on de Finetti's view of probabilistic semantics as grounded on the notion of coherence. Two papers (Bradley and Schulz) focus on the relationship between probability and counterfactual conditionals. Finally, the paper by Crupi and Iacona uses the notion of probabilistic relevance to capture the relationship between antecedent and consequent of a conditional.

The issue opens with Bradley's paper, which presents a range of suggestions within the framework of the suppositional conception of conditionals. The most intriguing, in my view, is the distinction between *evidential* suppositions and *interventional* suppositions. Evidential suppositions are hypotheses that, if true, should be added to the totality of the accepted information for determining the degrees of belief or epistemic probability. Interventional suppositions are assumed to be true as if they were due to an intervention external to the set of causal relationships that naturally hold among the events under consideration. Thanks to this distinction, Bradley can give a new characterization of the distinction between indicative and subjunctive (also called *counterfactual*) conditionals. Roughly speaking, the antecedent is an evidentiary supposition in indicative conditionals, while the antecedent is an interventional supposition in subjunctive conditionals. This approach allows Bradley to maintain that Adams' Thesis (that the probability of a conditional is the conditional probability of the consequent given the antecedent) applies to both indicative and subjunctive conditionals. This tenet suggests a unified suppositional view of indicative and subjunctive conditionals, which Adams rejected.

Schulz's paper shows that fresh problems arise about conditionals and probability if we also consider the notion of knowledge. He discusses a puzzle originally introduced by Rothschild and Spectre, and shows that it may be split into two parts, each of which can be resolved separately by the application of a single solution, thus proving the intimate connection among conditionals, probability, and knowledge.

Crupi and Iacona start from the idea (recently proposed by Bouven) that a conditional statement  $C$  if  $A$  is the more assertable, the more its antecedent  $A$

provides evidential support to the consequent  $C$  so that it *increases* its probability. This view is at odds with Adams' Thesis, according to which the degree of assertability of a  $C$  if  $A$  is just the conditional probability  $Pr(C | A)$  of the consequent  $C$  given the antecedent  $A$ , irrespective of whether  $A$  supports  $C$  or not. Crupi and Iacona define a new probabilistic logic with its probabilistic syntax and semantics. Their semantics defines a validity notion that shares some properties with Adams' p-entailment but diverges from it in other respects. For example, like Adams' language, their language does not contain compounds of conditionals, but unlike Adams', it has a recursive formation rule for the negation connective. The authors compare their logic with Adams' logic in detail, arguing that theirs is better suited in modelling a logic for indicative conditionals than is his (or, indeed, Douven's). The reader will assess this theory for herself, but, in any case, it deserves very serious consideration.

Nulvesu's paper represents a transition between the foregoing papers and those that follow and that are focused on the logic of de Finetti's tri-events. The paper aims to put de Finetti's theory of tri-events (dating from the first decades of the 20th century) in the context of the contemporary debates about conditionals. Nulvesu's paper is also useful to readers looking for an overview of the contemporary debates about conditionals, including Stalnaker-Lewis truth-conditionals views and Adams' views.

De Finetti's ideas permeate the paper by Gilio and Sanfilippo. However, they depart from de Finetti's truth-conditional tri-events and take a different direction, based on the de Finettian notion of coherence, namely, the idea that propositions or events are *random quantities* and conditional expectations. On this picture, it is not clear what the values 1 and 0 represent. They may be understood as genuine truth-values (albeit in non-realistic terms), as in the theory of tri-events, or as simply "values" or, to use Adams' phrase, as "ersatz truth-values". Gilio advanced the core ideas on which the present contribution is based in joint authorship with Romano Scozzafava.<sup>1</sup> The fundamental notion is that of a bet. However, de Finetti himself defined coherence in that context and extended it to random variables in general. De Finetti also introduced the idea of a conditional bet on an event  $C$  given an event  $A$ , characterized as a bet such that the bettor wins if both  $A$  and  $C$  occur, loses if  $A$  occurs, but  $C$  does not, and is void (or null) if  $A$  fails to occur. If the fair betting quotient of this bet is  $p$ , so that the fair price of the offer "1 if  $A$  and  $C$ , 0 if  $A$  and not  $C$ , and  $p$  if not  $A$ " has expected value  $p$ , Gilio and Scozzafava took the real number  $p$  as the (quasi-semantic—the qualification is mine) value of the conditional event  $C | A$ .

Gilio and Sanfilippo have developed this idea much more fully, but unfortunately, philosophers have little knowledge of their valuable work, despite the great interest that it presents from the philosophical point of view. Therefore, the publication of the essay in this issue aims also to disseminate their work among philosophers given its focus on the aspects that are most relevant to the debate on conditionals: compounds of conditionals and iterated conditionals. Their theory satisfies in a general way the Adams Thesis. So it escapes *Lewis' Triviality Results*. The theory of Gilio and Sanfilippo attributes a finite number of values to

<sup>1</sup> In the same year (1994), independently, Robert Stalnaker and Richard Jeffrey proposed a similar approach, although within a Kripke-style model semantics (cf. Stalnaker and Jeffrey 1994).

a conditional. In principle, this theory deals with conditionals of any complexity, even if the number of quasi-semantic values of a complex sentence grows with the number of distinct atomic sentences it contains. The probability of a complex event remains the expected value of its quasi-semantic values. A very remarkable feature of this theory is that its conjunction connective for simple conditional events satisfies, for  $p$ -entailment, both the introduction and elimination rules, which is a property that none of the truth-conditional theories satisfying the Adams' Thesis can satisfy.

I have qualified the values of Gilio-Sanfilippo conditional events as random quantities as 'quasi-semantic' so that the values 1 and 0 turn out to be ersatz truth-values rather than genuine truth values. Thus, there is no truth-conditional theory that supports the truth-conditional interpretation. From a philosophical point of view, one must consider the Gilio-Sanfilippo theory as a generalisation of Adams's theory, which postulates that conditional events (except the degenerate case in which they come down to ordinary events) are devoid of truth-value but have a degree of assertability, expressed by their probability. Of course, when only simple conditional events are involved, the Gilio-Sanfilippo theory coincides with Adams's theory.

Baratgin's paper is fundamentally a very important historical study of the origin of de Finetti's theory of tri-events. Baratgin's research antedates de Finetti's introduction of tri-events to 1928, while it was generally supposed that de Finetti invented them in the 1930s. *Prima facie*, this seems to be just a historical detail. It is not. To understand why one has to recall that the late Richard Jeffrey, after reading de Finetti's *Probabilismo* (1931) maintained that, *at that time*, de Finetti was, like himself, a *radical probabilist*. Radical probabilism is the doctrine according to which probability is a *primitive notion* and that one may characterise it without appealing to events that are considered certain. So, according to this doctrine, probability comes before truth. Was de Finetti a radical probabilist? In *De Finetti's Radical Probabilism*, Jeffrey writes:

De Finetti's probabilism is "radical" in the sense of going all the way down to the roots: he sees probabilities as ultimate forms of judgment which need not be based on deeper all-or-none knowledge (Jeffrey 1993: 264).

I discussed the matter with the late Horacio Árló Costa via email and face-to-face in 2011 in Konstanz. According to Árló Costa "this view is mistaken. There is plenty of evidence in de Finetti's writings that de Finetti did appeal to background certainties, which he used to define the notion of possibility. Finally, probability is distributed over this 'field of possibility'". Since both Jeffrey's and Árló Costa's views are well supported by textual evidence, in continuing our discussion, Árló Costa and I agreed on a reading of de Finetti as a radical probabilist when he wrote *Probabilismo*. However, we concluded that he changed his mind later, thus following Jeffrey's reverse intellectual pathway (who had abandoned the positions of Carnap to reach his radical probabilism). Now, de Finetti composed *Probabilismo* in 1928. Baratgin's paper shows that de Finetti elaborated his theory of tri-events in that year. Since this theory is based on truth-values and inspired by the betting situation, where the outcome is the paradigm of a certain event that decides whether the bet is won or lost, rethinking this matter, I concluded, in the light of Baratgin's paper, that this puzzle might be solved in the face of all the aspects of the complex de Finetti's epistemology

(albeit not explicitly elaborated). His sophisticated philosophy was influenced by Hume, Italian pragmatism, instrumentalism, operationalism, Machian philosophy, Vaihinger's "als ob" philosophy, and logical positivism. However, one can identify his epistemology with none of these views. Indeed, reading *L'invenzione della verità* (a philosophical essay that de Finetti wrote as a reaction to Carnap's *Der logische Aufbau der Welt* [1928]), it appears clear that one may combine the two conflicting views in the light of these considerations. De Finetti was a radical probabilist for all his scientific life. However, his radicalism is compatible with a constructive attitude to building a fairly idealised theory that gives rise to abstract notions, including truth and probability and the notions of event and conditional event or tri-event. This view is strongly influenced by Carnap's *Aufbau*, although his conclusions are completely at odds with Carnap's view, according to which the building of the "world" and knowledge rests on a solid foundation:

We see that everything is built on quicksand, although obviously, one tries to place the pillars on the relatively less dangerous points (de Finetti [1933] 2006: 145, English translation is mine).<sup>2</sup>

I hope to explain in more detail my reconstruction of de Finetti's complex epistemological views elsewhere. I want to emphasise here that Baratgin's paper sheds much light on this topic, suggesting that de Finetti never changed his views throughout his scientific life, especially on probability, events, and tri-events. Moreover, Baratgin's paper usefully compares de Finetti's theory with some more recent theories and developments.

My own paper presents a modified version of the truth-conditional theory of tri-events. I begin with the consideration that one cannot equip de Finetti's original theory with a notion of logical consequence in agreement with Adams' logic. Earlier attempts to reconcile tri-events and Adams' logic, proposed by myself or others, all suffer from the defect of having recourse to modal conditions in the definition of logical consequence. As a result, valid formulas cannot be instances of general schemas formulated by metalinguistic variables. Furthermore, those attempts inevitably attribute a special character<sup>3</sup> to basic or atomic statements, a remnant of Wittgenstein's *Tractatus* that de Finetti resolutely rejected (and that I, too, reject).

The semantics presented here is free from these difficulties. However, throughout a modal theory (formulated in Kripke's style), it is more exactly a generalization of a partial logic version of the S5 system. It extends Adams' theory to iterated and compound conditionals of any complexity so that logical consequence and Adams' *p*-entailment always coincide. The main philosophical aim of my contribution is to refute Adams' view that indicative conditionals al-

<sup>2</sup> De Finetti's views seem more like Popper's view as expressed in *The Logic of Scientific Discovery*: "Science does not rest upon solid bedrock. The bold structure of its theories rises, as it were, above a swamp. It is like a building erected on piles" (Popper [1935] 1992: 94).

<sup>3</sup> Cf.: "If we do not choose to ignore the way in which *S* has been derived from the basis *B*, the possibility arises that we could single out certain events as being somewhat *special*: for example belonging to the basis, or logically expressible in terms of a finite or countable number of basic elements" (de Finetti [1970] 1975: 271).

ways lack truth-values. My theory also satisfies Adams' equation so that it bypasses *Lewis' Triviality Results*.

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#### References

- Carnap, R. 1928, *Der logische Aufbau der Welt*, Berlin-Schlachtensee: Weltkreis.
- de Finetti, B. 1931, *Probabilismo: Saggio critico sulla teoria delle probabilità e il valore della scienza*, Napoli: Perrella.
- de Finetti, B. (1933) 2006, *L'invenzione della verità*, Milano: Cortina.
- de Finetti, B. (1970) 1975, *Theory of Probability*, Vol. 2, Chichester: Wiley.
- Jeffrey, R. 1993, "De Finetti's Radical Probabilism", in Monari, P. and Cocchi, D. (eds.), *Probabilità e induzione. Induction and Probability*, Bologna: CLUEB, 263-75.
- Popper, K.R. (1935) 1992, *The Logic of Scientific Discovery*, London: Routledge.
- Stalnaker, R. and Jeffrey, R. 1994, "Conditionals as Random Variables", in Eells, E. and Skyrms, B. (eds.), *Probability and Conditionals: Belief Revision and Rational Decision*, Cambridge: Cambridge University Press, 1994, 31-46.