Towards a Unified Theory for Conditional Sentences

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Abstract

A unified shared theory of conditionals does not still exist. Some theories seem suitable only for indicative but not for counterfactual ones (or vice versa), while others work well with simple conditionals but not compound ones. Ernest Adams' approach—one of the most successful theories as far as indicative conditional are concerned—is based on a reformulation of *Ramsey's Test* in a probabilistic thesis known as "The Equation". While the so-called *Lewis' Triviality Results* support Adams' view that conditionals do not express genuine statements, the problem arises whether these results lead inevitably to Adams' view—according to which conditionals always lack truth-values—or to the less radical view by Dorothy Edgington—according to whom *simple* (indicative) conditionals have well-defined truth-values only when they are used to make assertions and their antecedent is true.

I will suggest that Alberto Mura's account—a refinement of de Finetti's theory of tri-events that fits Adams' logic and extends it over the lattice of compound conditionals—can be a suitable candidate for a proper semantics of indicative conditionals and might be an interesting step towards a unified theory for conditional sentences.

Keywords: Indicative conditionals, Compound conditionals, de Finetti's tri-events, Adams' logic of conditionals, Lewis' Triviality Results.

1. Introducing Conditional Sentences

Conditional statements have been the subject of several discussions since ancient age. Indeed, linguistic constructions like "If *p*, (then) *q*" have always interested many philosophers and logicians because of their central role in common reasoning: every day we think and act under conditional statements. Unfortunately, the use of these sentences in theoretical and practical reasoning is quite tricky, often leading to absurdities:

[1] "If Trump dies, Biden will win the Elections. If Biden wins the Elections, Trump will resign immediately after the Elections. Therefore, if Trump dies, Trump will resign immediately after the Elections".

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[2] "If Tom were not at home, the lights would be off. The lights are on. Therefore, Tom must be at home".

Argument [1] represents a classical transitive schema whose premises may be both plausible while its conclusion is surely false, although it is a valid instance of a deductively valid argument. Argument [2] is a typical non-inclusive theoretical reasoning that might be easily invalidated by the additional information that sometimes Tom forgets to switch off home lights.

No less problematic is the use of conditional statements in practical reasoning:

[3] "I have heart disease. If I take medicines, I decrease the odds of a heart attack. So, I should take medicines".

Looking at example [3] from another perspective, it seems that people taking those medicines could have a heart attack easier than others. Misunderstanding like this could mislead the decision-maker! So, it is very important to pay attention to the action every conditional is affecting.

In common language, we deal with different types of conditional statements: simples, compounds, indicatives and counterfactuals. We call simple conditionals those sentences of the form $p \rightarrow q$ where "→"[1] does not occur neither in p nor in q and compound conditionals those compound sentences containing occurrences of conditional connectives in some of its proper sub-sentences. All sentences above are simple conditionals and also those conditionals whose antecedent or consequent contains a connective other than "→" (for instance '&', '∨', '⊃', '~') belong to this category.

[4] "If Robert buys the eggs and does not break them, then I will make a cake and take it to my mother" [$p \,\&\, q \rightarrow z \,\&\, v$]

[5] "If I win the lottery or inherit a fortune, then I will buy either a villa in Sardinia or an apartment in New York" [$p \vee q \rightarrow z \vee v$]

[6] "If it is a beautiful day, then if I find a ride, I will go to the beach" [$p \rightarrow (\mathrm{q} \rightarrow z)$]

Only sentence [6] is a compound conditional, while [4] and [5] are simple conditionals.

Regarding the difference between indicative and counterfactual conditionals, probably there is no better way than the following example to understand it:

[7] "If Oswald did not shoot Kennedy, someone else did" (*Non-counterfactual or indicative conditional*)

[8] "If Oswald had not shot Kennedy, someone else would have" (*Counterfactual or subjunctive conditional*)

This situation is a paradigmatic illustration because the first proposition is unquestioned, and the second is typically denied.[2] Indeed, unless we are not any conspiracy theorist, we can reject [8] despite accepting [7]. Another dissimilarity—although not crucial—is that [8] presents a modal aspect, namely a *necessary link* (logical or causal) between the antecedent and the consequent, which seems to be missing in [7]. So, the distinction between indicative and counterfactual

[1] Generally conditional connective is represented by '→', but we can find other symbols too ('⊃', '>', etc.), according to different interpretations. I assume that the conditional symbol '→' is not a truth-function, and in particular that it is different from the material conditional '⊃'.

[2] Adams 1970: 89-94.

conditionals is unquestionably pointed out by this example, at the expense of those aspiring to a unified theory simply denying this difference.[3]

We must clarify why counterfactual conditionals are usually identified with subjunctives and non-counterfactuals with indicatives, although there is no complete coincidence.

First, we say that a conditional statement is a *counterfactual conditional* when its antecedent is false.

Second, we say that a conditional statement is a *subjunctive conditional* when the English grammar requires 'would' in the main clause and past tense in the if-clause. It can happen that sometimes these different properties—interpretative and morphological—do not coexist at all so that some subjunctive conditionals do not exclude the possibility of a true antecedent:

[9] "If Chris went to the party this evening, and she probably will go, Tom would be enthusiastic.

In the same way, it may be possible to use indicative conditionals even if we know the antecedent is false:

[10] "If he is handsome, then I am Naomi Campbell!"

However, many philosophers hold it would be wrong to describe a counterfactual merely as a conditional whose antecedent is false. Rather, it would be better to identify it as a proposition that *invokes* in some way the antecedent's falsity.[4] Indeed when we say:

[11] "If Jones were present at the meeting, he would vote for the motion"

instead of:

[12] "If Jones is present at the meeting, he will vote for the motion"

we are pointing out a piece of information rather than another one: with [11] the speaker wants to focus the attention on what Jones would do if he were present at the meeting—without excluding the fact that he could *not* be present (so *invoking the antecedent's falsity*)—instead of [12] it is not important that part of the content about Jones' presence (or absence) but, rather, the information concerning the fact he intends to vote for the motion.

Let me present another example:

[13] "If I went to the prom, would you come with me?"

[14] "If I go to the prom, will you come with me?"

In front of these two utterances, the first thought is that saying [13] is trying to invite me to the prom—he says he would like to go to the prom with me. Instead, about [14] I could think that the guy (maybe a neighbour) is offering me just a

[3] "Therefore, there really are two different sorts of conditional; not a single conditional that can appear as indicative or as counterfactual depending on the speaker's opinion about the truth of the antecedent" (Lewis 1973: 3).

[4] "It is not their [the antecedent's and consequent's] falsity in fact that puts a 'counterfactual' conditional into this special class, but the user's expressing in the form of words he uses, his belief that the antecedent is false" (Mackie 1973: 71).

"'Counterfactual' may seem to be less open to objection. What lies behind this piece of terminology is not, of course, that the antecedent is in fact false, but that, in some way, the falsehood of the antecedent is implied, whether the conditional is true or false, well supported or not" (Woods, Wiggins and Edgington 1997: 5).

ride to the party (maybe by car)—I should be self-confident to think this utterance means a romantic date.

In other words, with [11] and [13] we want to remark just that necessary link between the antecedent and the consequent characterizes, as formerly said, the counterfactual conditionals rather than the indicative ones. Therefore, if we do not strictly denote counterfactuals with those conditionals whose antecedent is false, [11] and [13] could be easily considered counterfactuals as much as the following conditionals:

[15] "If Jones had been present at the meeting, he would have voted for the motion".

[16] "If I had gone to the prom, would you have come with me?"

Furthermore, if we only accepted sentences like [15] and [16] as counterfactuals and rejected [11] and [13], we should consequently treat the last ones such as contrary-to-facts. In this way, we would end to confuse the two well-defined classes. One must establish their differences, and a conditional does not have to work as a supporter for the other one.[5] Examples [7] and [8] entirely show this idea by examples: people who accept [7] hardly hold [8]. Instead, a person could easily accept both [11] and [15]—such as [13] and [16]—recognizing in them the same counterfactual conditional in two different times.[6] So, in order to facilitate, many philosophers—and I agree—have decided to deal with, in general, subjunctive conditionals as counterfactuals and indicative conditionals as non-counterfactuals.

In short, there are different types of conditional statements and to deal with all of them is not problem-free. It is not exhaustive to identify "If p, (then) q" simply with a sentence characterized by a link between an antecedent (p) and a consequent (q). A theory of conditionals must show the great importance of conditional statements when they are acceptable and true or simply assertive. For certain, this is not an easy task, and, although this field has made much progress, a genuinely unified theory of conditionals does not exist yet. Indeed, some theses seem good only for indicative and not for counterfactual conditionals (or vice versa) while others work well with simple conditionals but not with compound ones. A so-called unified theory should apply to all of these different accounts of conditionals.

2. The Equation and Adams' Thesis

Let us consider the famous remark—and footnote related—in Ramsey 1929 and some different suppositional theories born as their interpretation:

> Now suppose a man is in such a situation. For instance, suppose that he has a cake and decides not to eat it because he thinks it will upset him, and suppose that we consider his conduct and decide that he is mistaken. Now the belief on which the man acts is that if he eats the cake he will be ill, taken according to our above

[5] "As has been recognized, what would count as strong, or conclusive, support for a non-counterfactual conditional would not support the corresponding counterfactual" (Woods, Wiggins and Edgington 1997: 7).

[6] Surely, an indicative conditional could become counterfactual with the time, but this is not a proper distinguishing feature, such as examples [4] and [5] shows—neither [4] correspond to [5] or it is [5] in a second moment. At most the indicative "If Oswald didn't kill Kennedy, someone else did" could correspond to some kind of counterfactual like "If Oswald hadn't killed Kennedy, Kennedy would be still alive".

> account as a material implication. We cannot contradict this proposition either before or after the event, for it is true provided the man doesn't eat the cake, and before the event we have no reason to think he will eat it, and after the event we know he hasn't. Since he thinks nothing false, why do we dispute with him or condemn him?
>
> Before the event we do differ from him in a quite clear way: it is not that he believes p, we $\bar{p}$; but he has a different degree of belief in q given p from ours; and we can obviously try to convert him to our view.[1] But after the event we both know that he did not eat the cake and that he was not ill; the difference between us is that he thinks that if he had eaten it he would have been ill, whereas we think he would not. But this is prima facie not a difference of degrees of belief in any proposition, for we both agree as to all the facts.
>
> [1] If two people are arguing 'If *p* will *q*?' and are both in doubt as to *p*, they are adding *p* hypothetically to their stock of knowledge and arguing on that basis about *q*; so that in a sense' If *p*, *q*' and 'If *p*, $\bar{q}$' are contradictories. We can say they are fixing their degrees of belief in *q* given *p*. If *p* turns out false, these degrees of belief are rendered void. If either party believes $\bar{p}$ for certain, the question ceases to mean anything to him except as a question about what follows from certain laws or hypotheses (Ramsey 1929: 246-47).

The procedure for evaluating conditional sentences described in this text is called *Ramsey's Test*. It inspired a suppositional analysis for conditionals, where 'If *p*, *q*' is interpreted as a hypothetical supposition that the antecedent *p* holds the believability of the consequent *q* under that supposition.

Like Ernest Adams and followers, some philosophers—focusing their attention on the concept of *degree of belief*—considered the remark above to apply probability logic[7] to conditional sentences. So, they interpreted Ramsey's Test via classical Bayesian conditionalization, inviting to measure the probability of "If *p*, *q*" by *conditioning* on *p*, identifying the probability of a conditional with the *conditional probability* on *q* given *p*.[8] This construal represents the reformulation of Ramsey's Test in a probabilistic thesis known as "*Equation*":

$P(p \longrightarrow q) = P(q \mid p)$ [where $P(p) > 0$][9]

[7] Ramsey's probability theory—called "logic of partial belief" by himself—is built on the idea that human beliefs cannot be based on an objective theory because they are connected to a whole set of epistemic attitudes through which people evaluate, choose and act. Ramsey did not mean to deny the existence of objective beliefs, but just to suggest to interpret human knowledge in terms of partial beliefs able to change in front of new evidences. The logic of partial belief wants to be just a way to calculate our beliefs such as subjective probabilities, establishing Bayes' theorem as the general rule to determine the probability update.

[8] Thomas Bayes was able to found an updating rule establishing how to adjust our *degree of belief* when we acquire new information. Indeed, the probability of any event *b* after learning that *a* is true (and nothing else) may be changed. How? The rule prescribed in Bayesian literature is to match the *posterior* probability of *b* ($P_{t1}(b)$) with the *prior* probability of *b* given *a* ($P_{t0}(b \mid a)$). This is just the *Bayesian conditionalization*—where $P_{t0}(b \mid a)$ is called *conditional probability*—and it can be formulated in this way: If $P_{t0}(a)>0$, then $P_{t1}(b) = P_{t0}(b \mid a)$.

[9] $P(p)>0$ because of *zero-intolerance* property of conditionals, according to whom if *p* has no chance of being true, there is not any conditional probability. In other words, nobody use a conditional sentence when know that the antecedent's probability is 0 (cf. Bennett 2003: 53-57).

Many philosophers and logicians advanced several proofs in support or against the Equation. A very important contribution is that of Adams, who had the worth of extending probabilistic logic to conditionals.

Since the mid-90s, Adams showed powerful arguments defending the probabilistic interpretation of Ramsey's Test, so that some philosophers started to talk about Ramsey-Adams Thesis.

Adams' analysis is restricted to *indicative* conditionals and started observing that propositional calculus's common use leads to fallacies when its application involves conditional sentences. So, the problem Adams raised concerns on how we have to use formal logic in conditional treatment. Indeed, he showed that many classically valid cases—in the sense that the premises cannot be true while its conclusion is false—are rejected (or at least doubtful) by common sense, leading to different kinds of fallacies.

Adams identified the trouble because when we deal with conditional statements, the term 'true' has no precise application. For this reason, he proposed to find a kind of validity that does not involve the notion of truth, with the intent to analyze conditional sentences from the point of view and not in terms of their truth conditions. So, he substituted the concept of classical validity with that of *reasonableness*, whose condition is:

> If an inference is reasonable, it should not be the case that on some occasion the assertion of its premises would be justified, but denial of its conclusion also justified (Adams 1975: 171).

So, while classical validity involves the notion of truth, the reasonableness concerns *justified assertability*, which is not a mathematical or scientific notion but rather a concept whose content is dictated by the assertion context. An assertion of a statement is justified if *what one knows* on that occasion gives us either the certainty or a high probability that the same statement will be true and win a bet on it. In the same way, denying that assertion is justified if we have either the certainty or a high probability that the statement will be false and the bet will be lost. Adams called the assertion *strictly justified* in case of certainty and *probabilistically justified* when the statement is just highly probable.

What about the assertion of "If *p*, *q*"? Adams converted the above notions in terms of conditional bets[10]—any bets on conditional statements—giving such a "betting" criterion of justification:

a. The assertion of a bettable conditional 'if *p* then *q*' is strictly justified on an occasion if what is known on that occasion makes it certain that either *p* is false or *q* is true; its denial either *p* is false, or *q* is false.
b. The assertion of a bettable conditional 'if *p* then *q*' is probabilistically justified on that occasion if what is known on that occasion makes it much more likely that *p* and *q* are both true than that *p* is true and *q* is false; its denial is probabilistically justified if it is much more likely that *p* is true and *q* is false than that *p* and *q* are both true.

[10] The notion of conditional bet was introduced first by de Finetti 1931. See also de Finetti 1937.

c. (Definition) The assertion and denial of a bettable conditional 'if p then q' are both vacuously probabilistically and strictly justified on an occasion if what is known on that occasion makes it certain that p is false (Adams 1975: 176-77).

Conditions for reasonableness of conditionals	$p \rightarrow q$	$\sim(p \rightarrow q) \equiv p \rightarrow \sim q$
Strictly justified	$\sim p \vee q \equiv P(\sim p \vee q)=1$	$\sim p \vee \sim q \equiv P(\sim p \vee \sim q)=1$
Probabilistically justified	$P(p \wedge q) > P(p \wedge \sim q)$	$P(p \wedge \sim q) > P(p \wedge q)$
Vacuously strictly and probabilistically justified	$\sim p \equiv P(p)=0$	$\sim p \equiv P(p)=0$

In the case of vacuous justification, one may assert the inference and its denial because the bet is not lost but just *called off*—according to the betting criterion. However, Adams pointed out that when we are sure the bet will be called off, we will not stake at all and we are asserting no indicative conditional. Indeed, in those cases in which we are sure about antecedent's falsity, we will use a subjunctive conditional—Adams' analysis does not address that.

Considering the notion of vacuous conditional, Adams reformulated the general condition for reasonableness of an inference saying that *it cannot be the case the assertion of its premises and the non-vacuous denial of its conclusion are both justified on the same occasion*.

Because of this notion of reasonableness, Adams showed that absurd cases classically valid—for example, the material conditional's fallacies—is not valid for the betting criterion of justification. Indeed, inferences like "If Brown wins the election, Smith will retire to private life. Therefore, if Smith dies before the election and Brown wins it, Smith will retire to private life" (Adams 1975: 166) are classically valid but not *reasonable*, because both the assertion of the premises and the negation of the conclusion are justified.

In Adams 1965, we can find an informal presentation of a reasonableness' criterion using the standard probability calculus. After solving several problems in conditional treatment, Adams concluded that this first analysis shows some critical limitation. For example, its application lacks with conditionals derived from suppositions and with compounds involving conditionals. So, he advanced the hypothesis that, maybe, assertable conditionals observe different logical laws, others from those of the standard propositional calculus.

Trying to overcome these limitations, in Adams 1966 (265-316) the original idea is formalized with some adjustment. First of all, the notion of "justified assertability" is now replaced by that of "high probability", and the criterion of reasonableness is consequently given simply substituting "true" with "high probability" in the definition of classical validity:

> an inference is *reasonable* just if its premises cannot have high probability while the conclusion has low probability (Adams 1966: 266).

Then, in Adams 1975, a consequence of this assumption is made explicit, introducing a technical term called "uncertainty" (u = 1– probability). So, in case of reasonable inference:

> [...] *the uncertainty of its conclusion cannot exceed the uncertainty of its premises* (where uncertainty is here defined as the probability of falsity [...]) (Adams 1975: 2).

The concept of uncertainty is fundamental in Adams because he could avoid any falsity concept in the definition of validity.

Therefore, in Adams's hypothesis, the strict connection between high probability and truth, characterizing unconditional statements, fails for conditionals.[11] Indeed, Adams advanced the idea according to which the probability of a conditional sentence should not be interpreted as the *probability of being true* but rather as *conditional probability*. Thus, Adams identified the Equation as a fundamental assumption of his analysis, making his thesis one of the most important arguments in defence of the Equation itself.

The idea that probability equals the probability of truth[12] entails that truth-conditional validity ensures reasonableness (*probabilistic-validity*) but, according to Adams, it holds *only* in case of factual propositions. Thus, if Adams' supposition is correct, the probability of a conditional cannot equal *in general* the probability it is true.

The fact that such a link between truth-conditional validity and p-validity holds for a factual proposition is easily demonstrable. For example, an inference like "It will either rain or snow tomorrow ($R \vee S$); it will not snow tomorrow ($\sim S$); therefore it will rain tomorrow (R)" (Adams 1975: 88) is classically valid when $R \vee S$ and $\sim S$ are true, and R is true too. Now, suppose that both $P(R \vee S)$ and $P(\sim S)$ equal 95% so that both $P(\sim R \& \sim S)$ and $P(S)$ are of 5%. Under these circumstances, the sum of the premises' uncertainties is 10%, thus $u(R) \leq 10\%$. This result means that, if the premises have *objective* probabilities of 95%, their conclusion has a probability of at least 90%, and this connection between objective probabilities and correct predictability makes that truth-conditional validity guarantees the probabilistic-validity.

The connection mentioned above cannot be shown in conditional sentences, and the truth-conditional validity lacks proof for reasonableness. For example, consider the conditional inference "If I eat those mushrooms, I will be poisoned" (Adams 1975: 89). The simple fact to not eat the mushrooms makes the inference materially true, but it is really difficult to say whether the assertion is right or wrong, so that the decision connected to it would be the best or the worst in terms of practical interest. Indeed, if the mushrooms are not poisoned, but delicious porcinis, and I decide not to eat them, my choice would not be right. This consideration confirms Adams' intuition, according to which the truth-conditional validity of a conditional inference does not prove its reasonableness—its probabilistic validity

[11] "The probability of a proposition is the same as the probability that it is true. [...] What we want to argue next is that there is a much more radical divergences between the two soundness criteria in application to inferences involving conditional propositions, which is ultimately traceable to the failure of the probability equals probability of truth assumption in application to conditionals" (Adams 1975: 2).

[12] $P(\phi \Rightarrow \psi) = \frac{P(t_1)t_1(\phi\&\psi)+\cdots+P(t_n)t_n(\phi\&\psi)}{P(t_1)t_1(\phi)+\cdots+P(t_n)t_n(\phi)}$

cannot be guaranteed by classical validity—so that the rule according to which probability is the probability of truth fails with conditional inferences. Why? According to Adams, the explanation is that when we assert a conditional, we do not express a probability of truth, but nothing more than a conditional probability. This approach should explain many phenomena, like the mushrooms' example, in an easier way than standard probability. This remark is the reason for which Adams' Thesis is also known as Probability Conditional Thesis—(PCT): $P(p \rightarrow q) = P(q \mid p)$—and its relation with the Material Conditional Thesis—(MCT): $p \rightarrow q = p \supset q = \sim p \vee q$—is fixed by the *Conditional Deficit Formula* (CDF):[13]

$$P(p \supset q) - P(p \rightarrow q) = [1 - P(p \supset q)][\frac{P(\sim p)}{P(p)}].$$

Even though sometimes—when CDF is low—conditional probability can be inferred by material conditional, such a rule shows why generally they do not coincide at all.[14]

Although Adams' logic works pretty well with indicative conditionals, his thesis presents some limits because it does not always hold in common language. For example, inferences like "If it is sunny, then if it is my day off then I will go to the beach"—$p \rightarrow (q \rightarrow z)$—are excluded by Adams, but they may be asserted ordinarily, equalizing inferences like "If it is sunny and it is my day off, then I will go to the beach"—$(p \wedge q) \rightarrow z$—by the *Law of Importation*.[15] Also, inferences joining a standard proposition and a conditional one, like "Either I will stay at home or if Jane calls me then I will go to the cinema"—$p \vee (q \rightarrow z)$—are rejected by Adams, though they are really common in natural language.

However, our language is full of complications representing a real argument for rejecting a logic that seems to work well under many aspects. Perhaps, also, for this reason, Adams' hypothesis met several supporters. One of the most important is Dorothy Edgington (1986: 6-7), whose contributions helped make Adams' thesis one of the most shared in conditionals' field. Her arguments support either the Equation either Adams' conclusion that to accept $P(p \rightarrow q) = P(q \mid p)$ doubtless means to deny any truth conditions for conditional statements.[16]

[13] Adams 2005: 1-11.

[14] That these two kinds of probability do not coincide could be shown by a lot of example. One of these could be found in Adams 2005: 1-2.

[15] Law of Importation: $[p \rightarrow (q \rightarrow z)] \rightarrow [(p \wedge q) \rightarrow z]$. Vann McGee presented an argument supporting the ejection of iterated conditionals, reporting that when we say "If p, then if q then z" we are not accepting an iterated conditional, but rather a conditional with conjunctive antecedent, because what we have in mind is the conditional belief expressed by $(p \wedge q) \rightarrow z$. See McGee 1985: 462-71.

[16] However, in "On Conditionals" (2007: 180) Edgington accepts the view that when used to make conditional assertions, simple indicative conditionals may be considered true if both the antecedent and the consequent are true and false if the antecedent is true and the consequent false. "There is nothing comparably straightforward to say when the antecedent is false". According to Edgington, in any case, "[t]he 'true, false, neither' classification does not yield an interesting 3-valued logic or a promising treatment of compounds of conditionals".

3. Stalnaker Semantics for Conditional Statements

Moving from Adams' hypothesis appear to be a good start. When Robert Stalnaker developed his theory in 1968—using Kripke's models' technical machinery—tried to develop a truth-conditional semantics for conditionals—primarily for counterfactuals and covering the indicatives conditionals[17]—satisfying Ramsey-Adams Thesis. Indeed, in front of quite flawed theories—like the material implication analysis—Stalnaker thought to study Ramsey's Test, even though making some adjustments or trying to generalize it—given that Ramsey referred only to situations in which the agent has no idea about the antecedent's truth-value.

According to Stalnaker, this is the procedure for assessing credence to a conditional statement:

> First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true (Stalnaker 1968: 41-55).

Once established belief-conditions for conditionals, how to fix truth-conditions? In this regard Stalnaker resorted to the Kripkean notion of a *possible world*, meant as the "ontological analogue of a stock of hypothetical beliefs" (Stalnaker 1968: 45) so that conditionals' truth-conditions for conditionals can be provided by an adaptation of truth-conditions settled by the possible world semantics:

> Consider a possible world in which A is true, and which otherwise differs minimally from the actual world. *"If A, then B" is true (false) just in case B is true (false) in that possible world"* (Stalnaker 1968: 45).

A theory of conditionals in terms of Kripke's models—developed independently of David Lewis—allows the transition from belief-conditions to truth conditions.

Then, Stalnaker built a probability system C2 by three steps, where each step represents a probability system itself, extension of the previous one.[18] By C2 Stalnaker developed parallelism between his semantics and the theory of conditional probability, showing that the theorems of C2 are nevertheless the valid sentences of Ramsey's Test.

Therefore, according to Adams' hypothesis about simple conditionals, Kripke's semantics works even though it yields some problems in the presence of compounds of conditionals. These problems are just the limit showed by the famous *Lewis' Triviality Result*, which shows that, if the probability of conditionals is the conditional probability $P(q \mid p)$ (as Adams guesses) and the probability of a sentence is always the probability of being true (as Stalnaker supports). There are sentences p and q such that the conditional "If p, q" whose probability of truth

[17] "The analysis was constructed primarily to account for counterfactual conditionals—conditionals whose antecedents are assumed by the speaker to be false—but the analysis was intended to fit conditional sentences generally, without regard to the attitudes taken by the speaker to antecedent or consequent or his purpose in uttering them, and without regard to grammatical mood in which the conditional is expressed" (Stalnaker 1976: 198).
[18] That argument could be found in Stalnaker 1970: 107-28.

coincides with $P(q \mid p)$ does not exist. In other words, the Triviality Result shows the incompatibility between the assumption that the probability of a proposition is *the probability it is true* and the *conditional probability* (Lewis 1981: 129-47), formalizing the divorce between Stalnaker's theory and the Equation.

4. Lewis' Triviality Result

In 1976 Lewis presented an argument, known as Triviality Result, showing the incompatibility between the assumption that the probability of a proposition is the probability it is true and the conditional probability. In such a way, the divorce between Stalnaker's theory and the Equation is formalized. There are many versions of the Triviality Result, but I prefer reporting the Lewis' original one.[19]

- Preliminaries:
  - Suppose we have a formal language containing at least the truth-functional connectives plus "→". Every connective could be used to compound any sentences in this language, whose truth-value is given in terms of possible worlds.
  - Define the conditional probability function in such a way:
    - $P(q \mid p) = P(q \wedge p) \mid P(p)$, if $P(p) > 0$.[20]
  - Assume the following standard probability laws:
    - $1 \geq P(p) \geq 0$.
    - If p and q are equivalent—both true at the same world—, then $P(p) = P(q)$.
    - If p and q are incompatible—both true at no world—, then $P(p \vee q) = P(p) + P(q)$.
    - If p is necessary—true in every world—, then $P(p)=1$.
  - Suppose to interpret "→" such that:
    - $P(p \rightarrow q) = P(q \mid p)$, if $P(p) > 0$, i.e. the probability of a conditional is its conditional probability.

    So that, if it holds, this holds too:
    - $P(p \rightarrow q \mid z) = P(q \mid p \wedge z)$, if $P(p \wedge z) > 0$.
- First Triviality Result:
  - Take $P(p \wedge q)$ and $P(p \wedge \sim q)$ both positive, so that $P(p)$, $P(q)$ and $P(\sim q)$ are positive too. Now we have:
    - $P(p \rightarrow q) = P(q \mid p)$ holds by $P(p \rightarrow q) = P(q \mid p)$.
    - $P(p \rightarrow q \mid q) = P(q \mid p \wedge q) = 1$ and $P(p \rightarrow q \mid \sim q) = P(q \mid p \wedge \sim q) = 0$ hold by replacing z with q or $\sim q$ in $P(p \rightarrow q \mid z) = P(q \mid p \wedge z)$.

[19] A simpler version is that by Blackburn. See Blackburn 1986: 201-32.
[20] If $P(p) = 0$ then $P(q \mid p)$ remains undefined. A truthful speaker considers permissible to assert the indicative conditional $p \rightarrow q$ just in case $P(q \mid p)$ is sufficiently close to 1, i.e. only if $P(q \wedge p)$ is sufficiently greater that $P(\sim q \wedge p)$. Lewis 1981: 129.

- For every sentence r, $P(r) = P(r \mid q) \cdot P(q) + P(r \mid \sim q) \cdot P(\sim q)$ holds by expansion.
- Taking r as $p \rightarrow q$, we have:
  - $P(r) = P(q \mid p)$, by $P(p \rightarrow q) = P(q \mid p)$.
  - $P(r \mid q) = P(q \mid p \wedge q) = 1$ and $P(r \mid \sim q) = P(q \mid p \wedge \sim q) = 0$, by $P(p \rightarrow q \mid q) = P(q \mid p \wedge q) = 1$ and $P(p \rightarrow q \mid \sim q) = P(q \mid p \wedge \sim q) = 0$.

  So:
  - $P(q \mid p) = 1 \cdot P(q) + 0 \cdot P(\sim q) = P(q)$ holds by substitution on $P(r) = P(r \mid q) \cdot P(q) + P(r \mid \sim q) \cdot P(\sim q)$.
- *First conclusions:*
  - If $P(p \wedge q)$ and $P(p \wedge \sim q)$ are both positive then the propositions are probabilistically independent—that is absurd, though no contradictory.
  - Assigning standard true-values to any couple of propositions p and q, it derives that $P(q \mid p) = P(q)$, i.e. the conditional probability equals the probability of the consequent.

  Consequently:
  - Any language expressing a conditional probability is a *trivial language*.

- Second Triviality Result:
  - Suppose that "→" is a probability conditional for a class of probability functions closed under conditionalizing, and take any probability function P in the class and any sentences p and q such that $(p \wedge q)$ and $P(p \wedge \sim q)$ are both positive. Proceeding as before, we have again:
    - $P(q \mid p) = P(q)$.
  - Take three pairwise incompatible sentences q, z and r such that $P(q)$, $P(z)$ and $P(r)$ are all positive. Replacing the disjunction $(q \vee z)$ with p, we have that $P(p \wedge q)$ and $P(p \wedge \sim q)$ are both positive, but $P(q \mid p)$ does not equal $P(q)$. This result means there are no such three sentences.
  - *Second conclusions:*
    - P is a *trivial probability function* that never assigns positive probability to more than two incompatible alternative, so fixing at most four different values: $P(q)$=1 and $P(p)$=1—determining that $P(q \mid p)$ =1=$P(q)$—, $P(q)$=1 and $P(p)$=0—so that $P(q \mid p)$ is an undefined number—, $P(q)$=0 and $P(p)$=1—determining that $P(q \mid p)$ = 0 =$P(q)$—, $P(q)$=0 and $P(p)$=0—$P(q \mid p)$ is undefined again.

    Consequently:
    - For every class of probability function closed under conditionalizing, "→" cannot be a probability conditional unless the class consists entirely of trivial probability functions.

- Given that a probability function represents a possible belief system, and some of such systems are not trivial, then indicative conditionals cannot be considered probability conditionals for the whole class of probability functions.
- There is no guarantee that the probability of a conditional equals the corresponding conditional probability for all possible subjective probability functions, i.e. it is not a general rule that the absolute probability of a conditional proposition equals the probability of its consequence on condition of its antecedent.

It is quite clear that the second Triviality Result logically entails the first one, the reason for which Lewis' argument is generally called just "Triviality Result".

Even if Stalnaker firstly agreed with the idea the probability of a conditional equals its conditional probability, in front of Lewis' Result he seems to give up the Equation and, in general, a suppositional view. Alternatively, Adams held his thesis inviting to consider conditionals, not as standard propositions, but as particular linguistic constructions always lacking truth values and conditions. However, I want to point out that Adams denied any truth-values and conditions for indicative conditionals only in 1975, after knowing the problems raised by Lewis' Triviality Result. Of course, Adams proposed to analyze conditional inferences in terms of probability since his first approach, but I think this is different from the total denial of truth conditions. I am not completely sure he would have advanced such a "drastic" solution if any Triviality Result would not have been possible.

Lewis himself, although not explicitly, disagreed with Adams' conclusion. He thought that "fortunately a more conservative hypothesis is at hand" (Lewis (1981: 137): Grice's theory. One may identify its conversational rules with those special rules useful to understand why assertability goes with conditional probability. So, Lewis suggested that we should start from something already known, rather than run into those complications Adams' hypothesis requires. For this reason, he adopted the material conditional's truth conditions, explaining the discrepancy between its probability of truth and its assertability by a Gricean implication. In a first moment, Lewis talked about a *conversational* implication, but then he opted for Jackson's theory, in favour of a *conventional* one (Lewis 1987: 151-56):

> An indicative conditional is a truth-functional conditional that conventionally implicates robustness with respect to the antecedent. Therefore, an indicative conditional with antecedent A and consequent C is assertable iff (or to the extent that) the probabilities $P(A \supset C)$ and $P(A \supset C/A)$ both are high. If the second is high, the first will be too; and the second is high iff $P(C/A)$ is high; and that is the reason why the assertability of indicative conditionals goes by the corresponding conditional probability (Lewis 1987: 153).

In any case, the real problem of Adams' conclusion concerns compound of sentences. Indeed, even if he was right, and conditionals with truth-valued antecedent and consequent would be governed only by assertability rules—different from standard probability rules—what about those conditionals compounded of conditional antecedent and consequent, lacking themselves of any value, condition and probability of truth? Adams should admit that the common idea is that we can know the truth-conditions of molecular sentences once we know their sub-sentences' truth-conditions. However, how could it be possible when sub-sentences lack truth-conditions? In that case, we need something different from those assertability rules, because, in front of this new evidence, they cannot show how compound sentences work. We need at least a new semantics containing special rules or anything else able to explain them.

5. An Alternative Way according to de Finetti

Though not without difficulty, Adams' approach represents one of the most successful theories of conditionals. However, the question I raise is this:

Does the Triviality Result lead to Adams' conclusion to deny any truth conditions and values for indicative conditionals?

In other words:

Might it exist an alternative way to avoid Lewis' Result following the Equation?

To answer such a question, I will introduce de Finetti's logic, a kind of three-valued logic, called "Logic of Tri-events", which seems to avoid the Triviality Result—even though it is no free from trouble.

Bruno de Finetti is known to be the founder—together with Ramsey, but independently—of the subjective interpretation of probability. He developed his analysis in terms of a betting system: probability is a special case of prevision corresponding to a bet's price. In case of a conditional bet, that is a gamble on a proposition q supposed that an event p happens, its price will equal the conditional probability of $q \mid p$, i.e. a conditional bet coincides with a suppositional conditional.[21]

According to de Finetti, a conditional bet on q supposed that p will be (i) win when either p either q are true, (ii) lost when p is true and q false, (iii) called off when p is false. Therefore, he suggested to assign to $q \mid p$ a truth-value just in case of win or loss, and to consider it null—neither true nor false—when the bet is called for. In such a way a conditional event appears as a three-valued proposition, called "tri-event".

In 1935, de Finetti proposed a kind of logic of conditional events, known as "Logic of Tri-events", consisting of a three-valued logic expressing the question concerning conditional probabilities.[22] The basic idea is that the act to assume a standard two-valued logic is just a conventional issue: propositions are not true or false because of *a priori* principle, but because we conventionally decided to call "propositions" those logical entities needing of a "yes" or "no" as the answer. However, if we agreed on assume three values, we could have an analogue of standard logic, but with more values, differing just in a purely formal way.

In the Logic of Tri-events, the third value is not, strictly speaking, a value like "true" or "false". We have to consider a third possible attitude that someone

[21] De Finetti made use of the notion of "conditional expectation"—$P(X \mid H) = P(X \wedge H)/P(H)$—that allows to interpret the conditional probability such as the expected conditional value of the prize of a conditional bet. This is important because Stalnaker & Jeffrey and McGee made the mistake to consider the value of a conditional bet such as the absolute expectation value of its prize, interpreting a called off bet such as zero profit. But, put in this way, in the Bayesian theory a called off bet is something which remains unchanged to positive linear transformations—there is not any zero equipped of an intrinsic value. In de Finetti's view the gain of a called off bet is indefinite.

[22] Bruno de Finetti 1935: 181-90.

can adopt toward a proposition when he doubts answering "yes" or "no". In other words, this third value is void—or *null*—and one can understand it as a gap. However, a null event is something different from an indeterminate event á la Lukasiewicz—whose truth conditions are *unknown*. Rather, de Finetti meant an event whose truth-values true or false are not satisfied. We can find several de Finetti's papers talking about this third value, and he has never changed his interpretation about that. It is particularly interesting the passage in which he identified a null event with an "aborted event":

> If a distinction results in being incomplete, no harm was done: it would mean that besides "true" and "false" events I would also have "null" events, or, so to speak, aborted events. As a matter of fact, it is sometimes useful to consider explicitly and intentionally from the very start such a "tri-event" (especially, as will be seen later, for probability theory). If, for instance, I say: "supposing that I miss the train, I shall live by car", I am formulating a "tri-event", which will be either true or false if, after missing the train, I leave by car or not, and it will be null if I do not miss the train.[23]

One may expand standard logic's truth-tables to include the null value in such a way:

p	q	$\sim p$	$p \vee q$	$p \wedge q$	$p \supset q$[24]	$q \mid p$
T	**T**	F	T	T	T	T
T	**F**	F	T	F	F	F
T	**N**	F	T	N	N	N
F	**T**	T	T	F	T	N
F	**F**	T	F	F	T	N
F	**N**	T	N	F	T	N
N	**T**	N	T	N	T	N
N	**F**	N	N	F	N	N
N	**N**	N	N	N	N	N

While conjunction and disjunction coincide with those proposed by Lukasiewicz' three-valued logic, conditioning is the new truth-function introduced by de Finetti. So, the real innovation consists just in the truth-functional conditioning connective "|".

According to de Finetti, such a kind of logic should help us to manage those troubles due to a two-valued analysis, with the advantage that one may translate every proposition in terms of standard logic—given that every tri-event is a simply formal representation of pairs of ordinary events.[25] Indeed, a return from the Logic of Tri-events to the standard two-valued logic is possible by introducing

[23] Translation by Alberto Mura of Bruno de Finetti 1934/2006: 103, in Mura 2009: 204.
[24] This material conditional is today known as "Kleene's strong material implication", because independently proposed later by Kleene in 1938. See Kleene 1938: 150-55.
[25] However, it should be pointed out that the algebra of such a pairs of ordinary events—isomorphic to the trievents' algebra—is not Boolean. It is rather a distributive lattice that does not admit a unique complement—it means it does not hold CEM.

two operations: *thesis* (*T*) and *hypothesis* (*H*).[26] *T*(*X*) means "*X* is true" and *H*(*X*) means "*X* is not null":[27]

X	*T*(*X*)	*H*(*X*)
T	T	T
F	F	T
N	F	F

The above truth-table shows it holds that $X = T(X) \mid H(X)$, i.e. every tri-event ϕ is true, given that it is not null. This result is known as "de Finetti's Decomposition Theorem".[28]

Given that every tri-event can be represented by any conditional event $q \mid p$—where *p* and *q* are ordinary events—, for the Decomposition Theorem it holds that $q \mid p = T(q \mid p) \mid H(q \mid p)$. Looking at the truth-table of "|", excluding those cases where *p* and *q* are aborted events, the Decomposition Theorem leads to two important consequences:

- $q \mid p$ is true if and only if both *p* and *q* are true—$T(q \mid p) = p \wedge q$.
- $q \mid p$ is not null if and only if *p* is a tautology—$H(q \mid p) = p$.

p	***q***	***q* | *p***
T	**T**	**T**
T	**F**	**F**
F	T	N
F	F	N

Thus, it results that $q \mid p = T(q \mid p) \mid H(q \mid p) = (p \wedge q) \mid p$.

If *p* is not a tautology, it means it could be false, so that $q \mid p$ is not an ordinary event. Consequently, an ordinary event is nothing less than a particular case of a tri-event when p is a tautology. Therefore, "to introduce the notion of conditional probability is to extend the definition of *P*(*X*) from the field of ordinary events, *X*, to the field of tri-events" (de Finetti 1935: 184-85). In Mura 2009 (214-16) we can find two methods to obtain such extension.

In conclusion, de Finetti's analysis shows that every probability function defined on a Boolean algebra of ordinary events can be univocally extended to the whole tri-events lattice, so that, given two standard proposition *p* and *q*, the probability of the tri-event $q \mid p$ equals the ratio between the probability of $p \wedge q$ and the probability of p. Consequently, "|" appears such as a connective satisfying the Equation but with the advantage of avoiding Lewis' Triviality Result—because $q \mid p$ is not an ordinary event, but a three-valued proposition.

[26] The rule of Thesis and Hypothesis is just that of allowing a conversion into standard logic. So, technically, they are not operators belonging to the logic of Trievents. About this, see Mura 2009: 207-209.
[27] In terms of betting system, "the 'thesis' of the tri-event, is the case in which one has established that the bet is won; the 'hypothesis' the case in which one has established that the best is in effect" (Bruno de Finetti 1935: 186).
[28] So called by Alberto Mura. See Mura 2009: 208.

Although de Finetti's account can represent a way to avoid trivialization conserving the Equation, it is not free from problems, making it unable to provide a right semantic for conditional statements. For example, despite increasing relation to some aspects, the correspondence between logic and probability loses some properties on the other side. Among them, in de Finetti's account, $\phi \mid \phi$ is not a tautology, but a *quasi-tautology*, because although it is not false, it can be either true or null. So, given any p and q and any probability function P, if $P(p) = P(q)$ but $p \mid p$ is not truth-functionally equivalent to $q \mid q$, then $p \neq q$. In other words, it does not work the propriety according to which, when two tri-events have, for every probability function, the same probability, the respective propositions are logically equivalent. This result means there are various tri-events, to which every probability function assigns probability 1, but not logically equivalent. Similarly, any de Finetti's contradiction is a *quasi-contradiction*,[29] given that it cannot be true, but can be either false or null. So, it is easy to catch that some elements can be quasi-tautologies and quasi-contradictions simultaneously.

Some of de Finetti's approach seems to be overcome by a modified tri-events approach, developed by Alberto Mura.

Mura's proposal was presented first as "Semantics of Hypervaluations" and then improved as "Theory of Hypervaluated Trievents". Mura gave a modified account of de Finetti's tri-events—escaping different arguments against the original tri-events—with the intent of providing a new semantic for Adams' conditional logic. I will claim that Mura's account can be a good candidate for a semantic of indicative conditionals, in perfect harmony with Adams analysis. Indeed, the Theory of Hypervaluated Tri-events incorporates Adams' p-entailment, allowing an extension of it for all tri-events—including compounds of conditionals. In this way, we are no more obligated to reject any truth conditions for conditionals.

Moreover, Mura proposed a generalization of the Theory of Hypervaluated Trievents to catch counterfactual conditionals by introducing a new variable K representing the corpus of total beliefs.

In conclusion, I wanted to evidence that conditional issue is not a closed topic and that different additional ways can be investigated. For example, any theory developed on a three-value logic might be a good solution that deserves to be inquired, letting us still aspire to a unified theory for conditional sentences.

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[29] Both terms of "quasi-tautology" and "quasi-contradiction" due to Bergman. See Bergman 2008: 85-86.

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