Reconsidering an Ontology of Properties for Quantum Theories

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Abstract

Da Costa, Lombardi and Lastiri (2013) have proposed an ontology of properties for non-relativistic quantum mechanics within the structure of the modal-Hamiltonian interpretation of the theory. Recently, this proposal has been developed in order to discuss the nature of entanglement and indistinguishability in such an ontology (Fortin and Lombardi 2022) and to explain how particles emerge from an ontology of properties (Lombardi and Dieks 2016). Oldofredi (2021) has also proposed an ontology of properties for Relational Quantum Mechanics. The aim of my paper is then to discuss an ontology of properties in the context of Quantum Field Theory (QFT). Wayne (2008) and Kuhlmann (2010) have already tried to define an ontology of properties for QFT in terms of tropes. However, I will try to follow a different approach. On the one hand, I will give a more general framework, which does not necessarily entail an ontology of tropes. I will in fact follow the path originally suggested by Da Costa, Lombardi and Lastiri, which is more general and formal. I will also briefly discuss how such an approach can be represented in an algebraic formalism. On the other hand, I will show how such an ontology of properties provides a good interpretation and representation of the measurement models that we can define in the context of QFT (by following the recent discussion of the measurement problem in QFT given by Grimmer 2022), and then of the experimental results.

Keywords: Ontology, Properties, Particles, Quantum theories, Superselection formalism.

1. Introduction

The aim of this paper is to discuss a possible ontology of properties for quantum theories (QTs). The idea was originally formulated by Da Costa, Lombardi and Lastiri (2013) and has been quite ignored until recent years, when the original proposal has been developed and eventually generalized in order to cover other aspects of QTs, which were not properly analyzed in the original paper (see, for example, Holik, Jorge, Krause and Lombardi 2021 and Fortin and Lombardi 2022). Moreover, several scholars have recently considered different proposals of bundle theories as possible interpretations of QTs. In particular, Oldofredi (2021)

Argumenta 8, 2 (2023): 427-442

ISSN 2465-2334
DOI 10.14275/2465-2334/202316.ros

has proposed a mereological bundle-theoretic interpretation of Rovelli's Relational Quantum Mechanics (see Rovelli 1996 for the original formulation of Relational Quantum Mechanics). I do not want to enter in that debate in this paper, but I consider such a proposal as a good example of how an ontology of properties for QTs is still interesting and worth discussing.

Da Costa, Lombardi and Lastiri (2013) propose an ontology of properties in the context of the modal interpretation of Quantum Mechanics (QM). In particular, such an ontology has been suggested for the modal-Hamiltonian formulation of QM, which has been formulated by Lombardi and Castagnino (2008) and which is a specific form of modal interpretation of QM that rests on the primary role that Hamiltonian formalism plays in the theory (I will come back to this point later in the paper). Yet, I think that Da Costa, Lombardi and Lastiri's proposal is interesting even if we do accept neither the modal interpretation of QM nor the specific modal-Hamiltonian interpretation, on which they ground their ontology of properties. Moreover, I think that Da Costa, Lombardi and Lastiri's proposal can be relevant also in the context of Quantum Field Theory (QFT), and of its algebraic formulation.

The search for an ontology in QFT has been one of the main problems for this theory. The main alternatives in the literature are, on the one hand, a particle ontology, where the fundamental ontological posits of the theory are particles. However, a series of results and no-go theorems seem to rule out such an ontology. The notion of particle should be in fact characterized by two fundamental features, that is, a particle should be a countable and localizable entity. However, the countability requirement is undermined by Haag's theorem and the Reeh-Schlieder theorem, which respectively prove that a unique total number operator for both free and interacting quantum field systems is not definable, and that a local number operator is also not definable either, if we take into account the mathematical structure of the theory (see Earman and Fraser 2006). Second, Malament's theorem proves that it is not possible to sharply localize particles in any bounded region of space-time; and hence also the localizability requirement does not seem to be satisfied (see Malament 1996). On the other hand, the natural alternative is an ontology of fields, but it turns out to be unworkable too. Baker (2009) shows that the Fock space formalism and the wavefunctional formalism are unitarily equivalent. This would mean that the problems of the particle interpretation—which is represented via the Fock space formalism—might be shared also by the field interpretation—which is represented by the wavefunctional formalism. In any case, one can try to resist these no-go theorems and arguments and try to provide a weaker notion of particle and of field that might avoid the problems above mentioned. For example, Fleming and Bennett (1989) and Fleming and Butterfield (1999) assume a notion of localization for particles that is relative to a hyperplane of simultaneity and, hence, immune to Malament's theorem—at the cost of losing the full relativistic invariance of the theory. However, these attempts to save a particle or a field interpretation of QFT do not seem eventually compelling.[1] There are also

[1] See also Fraser 2008 and 2017 and Rossanese 2021 for what concerns the notion of particle and the possible particle interpretation of QFT. See also Kuhlmann 2012 for what concerns the notion of field (and the possible field interpretation of QFT) and for a gen-

other possible ontologies for QFT, but each of them faces some problems once the formalism of the theory is considered. I will not enter in that debate here, but the interested reader can look at Kuhlmann, Lyre and Wayne 2002, where different ontologies for QFT are proposed and discussed (see also Kulmann 2012 for a complete presentation of the philosophical problems of QFT and for some references to how such a debate on the ontology of QFT has developed).

In any case, I think that Da Costa, Lombardi and Lastiri's original proposal can be properly generalized in order to be applied also to QFT. An ontology of properties has been already suggested for QFT, but I believe that Da Costa, Lombardi and Lastiri's ontology is, in a sense, more general and has less metaphysical costs. In fact, Wayne (2008) and Kuhlmann (2010) propose an ontology of particularized properties, that is, of tropes in order to interpret QFT. However, such proposals face some problems that undermine their validity (see Rossanese 2013). Da Costa, Lombardi and Lastiri's ontology of property is, therefore, a good candidate to provide an ontology of QFT or, at least, is worth discussing in the context of this theory. There is surely more work to do in order to have a full and compelling ontology of properties for QFT, but this paper aims to pose the first brick.

The paper is then structured as follows. The second section will present and discuss Da Costa, Lombardi and Lastiri (2013)'s original proposal. The third section will try to generalize their proposal to the formal context of QFT (and in particular to the algebraic reformulation of the theory). Finally, in a fourth and conclusive section, I will provide some comments and a possible "to do list" for the future work toward a proper ontology of properties for QFT.

2. An Ontology of Properties for Quantum Mechanics

Da costa, Lombardi and Lastiri (2013) adopt a modal interpretation of QM and then provide their ontology of properties within that interpretation. First of all, we therefore need to have a look at what this interpretation assumes and, in particular, to the specific modal-Hamiltonian interpretation, which has been formulated by Lombardi and Castagnino (2008).

There is not a unique modal interpretation of QM, but rather there are several possible modal interpretations of this theory; each of them, however, share the same fundamental posit: "the state delimits what can and cannot occur, and how likely it is—it delimits possibility, impossibility, and probability of occurrence—but does not say what actually occurs" (van Fraassen 1991: 279). Then, the fundamental idea at the basis of the modal interpretation is to consider that quantum states prescribe only the possibility for a quantum event (that is, a measurement) to occur. Moreover, as Da Costa, Lombardi and Lastiri (2013: 3673) recognize, all modal interpretations share some specific features: (i) they assume the standard formulation of QM, (ii) they are realist interpretations of QM, (iii) as mentioned, the quantum state describes possible properties and their corresponding probabilities, which evolve according to the Schroedinger equation, (iv) a quantum measurement is a physical process, and finally (v) the quantum state describes a single system.

eral discussion of different possible answers to the no-go theorems presented in this section.

The aspect that is different in each modal interpretation is the specific rule of actual-value ascription. Each modal interpretation has, in fact, its own rule to ascribe actual values to the possible properties that are described to the quantum state. The ascription of actual-valued properties depends on the choice of the preferred context, which defines "the set of the observables that acquire an actual value without violating the restrictions imposed by the contextuality of quantum mechanics (Kochen and Specker 1967)" (Da Costa, Lombardi and Lastiri 2013: 3673).

Da Costa, Lombardi and Lastiri then adopt the specific modal-Hamiltonian interpretation of QM, which ascribes a fundamental role to the Hamiltonian of the quantum system. According to Lombardi and Castagnino (2008), in fact, the Hamiltonian of the quantum system plays a central role in the definition of what a quantum system is (and consequently what a quantum subsystem is). Moreover, it is also important in the choice of the preferred context, and then to the definition of a specific rule of actual-value ascription. It is then possible to define both a quantum system and a composite quantum system as follows (Da Costa, Lombardi and Lastiri 2013: 3673-374; see also the original formulation of Lombardi and Castagnino 2008).[2]

Systems postulate: A *quantum system S* is represented by a pair *(O, H)*such that (i) O is a space of self-adjoint operators on a Hilbert space H, representing the observables of the system, (ii) $H \in O$ is the time independent Hamiltonian of the system S, and (iii) if $\rho_0 \in O'$ (where O' is the dual space of O) is the initial state of S, it evolves according to the Schrodinger equation in its von Neumann version.

Composite systems postulate: A quantum system represented by S: (O, H), with initial state $\rho_0 \in O'$, is *composite* when it can be partitioned into two quantum systems S^1: (O^1, H^1) and S^2: (O^2, H^2) such that (i) $O = O^1 \otimes O^2$, and (ii) $H = H^1 \otimes I^2 + I^1 \otimes H^2$, (where I^1 and I^2 are the identity operators in the corresponding tensor product spaces). In this case, the initial states of S^1 and S^2 are obtained as the partial traces $\rho_0^1 = Tr_{(2)}\, \rho_0$ and $\rho_0^2 = Tr_{(1)}\, \rho_0$; we say that S^1 and S^2 are *subsystems* of the composite system, $S = S^1 \cup S^2$. If the system is not composite, it is *elemental.*

So far, we have defined the quantum system and the composite quantum system in terms of the Hamiltonian formulation of QM. As said, quantum states identify the possible properties of a quantum system. Then, in order to have actual-valued properties, we have also to define an actualization rule that helps us to identify a preferred context.

[2] It is important to note that Lombardi and Castagnino (2008) adopt an algebraic formulation of QM and this will be important when I will discuss the generalization of Da Costa, Lombardi and Lastiri (2013)'s ontology of properties to QFT, and in particular to its algebraic formulation. It is also worth mentioning that the algebraic perspective might be considered as the formal representation of the logical priority of properties over objects (as particles and fields), since in an algebraic formulation of QTs, the observables are the fundamental entities, while states are secondary, namely functionals over the algebra of observables. I will discuss this point in the third section of the paper.

Actualization rule: Given an elemental quantum system represented by S: (O, H), the actual-valued observables of S are H and all the observables commuting with H and having, at least, the same symmetries as H.

This actualization rule forbids to assign actual values to all those observables that possess fewer symmetries than the Hamiltonian, and therefore gives us a preferred context in terms of the symmetries of the Hamiltonian. As we shall see, the identification of such symmetries is extremely important and plays a fundamental role also in the possible generalization of this proposal to QFT. In any case, this is the modal-Hamiltonian interpretation of QM in a very brief and sketchy presentation.

Da Costa, Lombardi and Lastiri consider this interpretation as the basis for their ontology of properties for QM and define three types of properties, as the fundamental elements of their ontology.

First of all, there are **type-properties**, which can be considered as universal properties. In the specific context of QM, type-properties are the observables or a set of observables and are mathematically represented by self-adjoint operators or a set of self-adjoint operators. Type-properties, then, represent possible properties of a quantum system such as the possibility of having a definite energy.

Second, there are **case-properties**, which are instances of type-properties. In a more detail, case-properties are the possible values of the observables and are mathematically represented by the eigenvalues of the self-adjoint operators. For example, if the type-property of a certain quantum system is energy, the relative case-property is the possible energy of "this" quantum system describing a free electron. That is, the energy of a free electron is an instance of a more general property such as the energy, and the case-property represents the possible values that such instance can take if a measurement is made.

Finally, there are the **actualized case-properties**, which are the actual values that we collect after a measurement of certain specific observables.[3]

Da Costa, Lombardi and Lastiri's proposal is then a three-fold ontology of properties for QM, which is grounded in the modal-Hamiltonian interpretation of the theory. However, as mentioned, any modal interpretation needs an actualization rule in order to specify the preferred context and give us the list of all the observables that can have actual values. In the case of the modal-Hamiltonian interpretation, such preferred context depends on the symmetries of the Hamiltonian itself. Then, the next step is to identify which are the relevant symmetries and which are the relevant type-properties and observables that we should consider.

Da Costa, Lombardi and Lastiri assume that the space-time is Galilean (that is, space is considered as homogeneous and isotropic and time is considered as homogeneous) and then that quantum systems are invariant under the Galilean group of transformations. The structure of the group of transformations helps us to identify which are the fundamental type-properties that can take actual values, and that can be then considered in our ontology (all the other (non-fundamental) type-properties can be derived from these fundamental type-properties). In fact, it is possible to consider as fundamental type-properties all

[3] It is important to notice that in such an interpretation, a measurement is just a physical process regarding an interaction between two physical systems, with no difference with other kinds of interactions.

those properties that are represented by the Casimir operators of the Galilean group of transformations. For, Casimir operators are the generators of the group that commute with all the generators of the group and are then invariant under all the transformations of the group. Such fundamental type-properties are mass (M), internal energy (W) and squared spin (S^2).[4]

This allows Da Costa, Lombardi and Lastiri to refine their actualization rule in the following terms:

> **[Actualization rule]′:** The only instances of universal type-properties that actualize are the instances [M^i] of the mass [M], [W^i] of the internal energy [W], [S^{2i}] of the squared spin [S^2], and the instances of the universal type-properties that, in each case, are represented by operators obtained as functions of the Casimir operators of the Galilean group (Da Costa, Lombardi and Lastiri 2013: 3679).

We finally have all the elements that constitute Da Costa, Lombardi and Lastiri (2013)'s ontology of properties for QM. We can therefore define the notion of a bundle as a collection of instances of type-properties (and consider that only the instances selected by the preferred context can actualize). A couple of brief considerations are in order before we can proceed. First, the notion of bundle is correlated with the physical notion of a "system", and thus helps us to take into account the possible composite nature of a physical system. Second, since the actualization rule depends on the specification of a preferred context, and given the phenomenon of quantum indeterminacy, it is important to notice that this is the reason why it is better to characterize the bundle by means of type-properties, rather than of actualized case-properties.[5]

Atomic bundles are then defined in terms of the irreducible representations of the Galilean group, where the Casimir operators are multiple of the identity, namely $M = mI$; $W = wI$; $S^2 = s(s+1)I$.

A bundle is **atomic** if it has the following features:

(1) It has no more than one instance of each type-property;

(2) Instances of M, W and S^2 belong to it;

[4] It is important to note that, to be more precise, M, W, and S^2 are the generators of the Bargmann group, which is the central extension of the Galilean group. It is also important to notice that these properties might have a different role in different formulation of the theory. In fact, it is possible to consider mass (M) as a non-dynamical parameter of the theory, while the internal energy (W) and squared spin (S^2) can be considered as dynamical observables. I would like to thank an anonymous referee for having stressed these two points. However, I think that in my proposal it is sufficient to identify those properties with the fundamental properties of the system, which are indeed invariant to the symmetry group transformations—and the technical aspects mentioned in this footnote do not seem to undermine that possibility. In any case, see Ardenghi, Castagnino and Lombardi 2009a and 2009b for a detailed discussion of the role of Casimir operators in the context of the Galilean group of transformations and of their role in the modal interpretation of QM.

[5] This is also an argument in favor of the preferability of such an ontology of (universal) properties, rather than of an ontology of tropes (such as Wayne 2008's and Kuhlmann 2010's), which, in a sense, can be considered as the actualized case-properties of Da Costa, Lombardi and Lastiri's proposal. See also Rossanese 2013 for a specific evaluation of a trope ontology for QFT.

(3) It is represented by observables, which are in turn represented by operators that are multiple of the identity.

For example, according to this ontological framework, an **elementary particle** is represented by the triplet (*m*, *w*, *s*). However, it is important to note that, in such an ontology of properties, particles are only emergent entities, while the fundamental ontological posits of the theory are the fundamental type-properties that belong to the bundle, which in turn represents the so-called "particle system" (see, for example, Dieks and Lubberdink 2011 and Lombardi and Dieks 2016 to have an idea of how it is possible to recover the notion of particle from an ontology of properties).[6]

To sum up this section, states encode the measure of the propensity to actualization for all the case-properties, which are all the instances of type-properties, which belong to the bundle. Moreover, in the specific formalism of the Modal-Hamiltonian interpretation of QM, states are then represented by expectation-value functionals over the algebra of observables, as we have briefly mentioned in footnote 2. Then, the fundamental entities are the observables, which represent type-properties, while states are secondary entities. As we shall see, this point will be important for the possible generalization of this proposal to QFT, and to its algebraic reformulation in particular.

3. An Ontology of Properties for Quantum Field Theory

So far so good for what concerns QM, but then the interesting challenge is to test the validity of Da Costa, Lombardi and Lastiri's proposal also in the context of QFT. I think that we have two options on the table, which are not actually separated. On the one hand, we can consider the original proposal and see which elements need to be substituted in order to grasp the physical content of QFT. On the other hand, we can consider the specific formalism of the Algebraic Quantum Field Theory (AQFT) and search for the formal structures that we need to represent our three-fold ontology of properties.

Starting from the original proposal, I think that it is possible to preserve the structure of Da Costa, Lombardi and Lastiri (2013)'s ontology of properties. Surely, it is possible to maintain three different types of properties, since their definition can hold also in the formal context of QFT. It is also possible to maintain the basic structure of Lombardi and Castagnino (2008)'s modal-Hamiltonian interpretation of QM. We must, however, consider the Poincaré group of transformations, rather than the Galilean group; and search for the Casimir operators of the Poincaré group (and of the relevant gauge group as well). In fact, the quantum systems described by QFT are not invariant under the Galilean group, since QFT assumes a Minkowski space-time. Then, we have to study the structure of the Poincaré group and of its representations. Moreover, we also need to consider gauge symmetries, which have a fundamental role in the description of quantum states in QFT.

[6] As we shall see, this ontological framework can be "easily" extended in the context of relativistic quantum theories as QFT. There is, however, a fundamental difference that concerns the symmetry group that has to be considered. In the non-relativistic case, in fact, we consider the Galilean group, while in the relativistic case we need to consider the Poincaré group.

Ardenghi, Castagnino and Lombardi (2009b) discuss the case of a QFT with *U(1)* gauge fields, such as Quantum Electro Dynamics (QED). In particular, they study the representations of Poincaré group and find the relevant Casimir operators for mass and spin. They also proved that it is possible to recover the Galilean group and its Casimir operators in the limit, via a (generalized) Inonu-Wigner contraction.[7] Of course, this analysis is, in a sense, in the same spirit of Wigner's analysis (1939), who proves that an elementary particle, that is, a fundamental particle, can be understood as an irreducible representation of the Poincaré Group. However, it is important to notice that here the notion of fundamentality cannot be ascribed to particles, but rather to the fundamental type-properties that are identified thanks to the study of the observables and of the Casimir operators of the relevant group of transformations. Then, from a technical point of view, such analyses are similar, but I think that they differ from a philosophical and interpretational perspective.

In any case, at this point, we have only considered the space-time symmetries, but we have already said that internal gauge symmetries play a fundamental role in the structure of QFT. With respect to this aspect of the theory, Ardenghi, Castagnino and Lombardi discuss the case of the Abelian Lie group *U(1)* in the context of QED, and show that the Casimir operator $C^{U}{}_{1} = Q$ of this group is an actual-valued observable of the *U(1)* gauge quantum fields. In particular, the operator Q of the internal gauge group *U(1)* is the charge operator $Q = eI$.[8] This means that it at least is possible to generalize Da Costa, Lombardi and Lastiri's proposal in the context of QFT, by maintaining the Lombardi and Castagnino (2008)'s formalism and change the relevant group of symmetries of the Hamiltonian in order to be consistent with the symmetry structure of QFT.

As mentioned, another important feature of Lombardi and Castagnino (2008)'s framework is that it takes at its basis the algebraic formulation of QM. I

[7] It is important to note here that it is true that the Inonu-Wigner contraction allows to recover the Galilean group from the Poincaré group. However, as said in a previous footnote, we are interested in the central extension of the Galilean group, namely the Bargmann group. In order to recover the Bargmann group, therefore, the fundamental idea is to consider the central extension of the Poincaré group and then apply the Inonu-Wigner contraction. Yet, the Poincaré group does not have non-trivial central extensions. For this reason, one has to apply a generalized limiting procedure—that is, a generalized Inonu-Wigner contraction—in order to recover the central extension of the Galilean group from a trivial central extension of the Poincaré group. I would like to thank again an anonymous referee for helping me to be more precise on these technical aspects. See Ardenghi, Castagnino and Lombardi 2009b, and in particular their 5.2. section, for the definitions of two interesting limiting procedures and for the detailed demonstration of the possibility to recover the central extension of the Galilean group and its Casimir operators in the limit.

[8] In the context of QED, we need a representation of electric charge and this is possible thanks to the charge operator Q. In order to apply our framework to QED, we then need to find the Casimir operator that is associated with Q, which would assure us that this observable is invariant under the group transformations and, hence, can be considered as a fundamental property of the theory. Such a Casimir operator exists and it is represented by $C^{U}{}_{1}$. See again Ardenghi, Castagnino and Lombardi 2009b for a detailed discussion of the role of Casimir operators in gauge theories.

think, then, we can learn something by the study of the algebraic formulation of QFT.[9]

The main important aspect of AQFT is to focus on the structure of the algebras of observables rather than to the observables themselves. The fundamental idea of AQFT is to define a net of C*-algebra[10] $O \rightarrow A(O)$ that associates a C*-algebra $A(O)$ to every open bounded region of Minkowski space-time O. A local algebra $A(O)$ represents what is localized and then observables in the region O. The physical content of AQFT is then encoded in the so-called net of local algebras, that is, the mapping $O \rightarrow A(O)$ from regions O of Minkowski space-time to algebras of local observables $A(O)$.[11]

According to the Haag-Kastler formulation of the theory, the net of local algebras has to satisfy four axioms, which impose important physical conditions on the abstract C*-algebra $A(O)$:[12]

(1) Isotony: the mapping $O \rightarrow A(O)$ is an inductive system. This means that an observable measurable in the region of space-time O_1 is a fortiori measurable also in a region of space-time O_2 containing O_1.
(2) Microcausality: if O_1 and O_2 are space-like separated space-time regions, then $[A(O_1), A(O_2)] = \{0\}$. That is, all observables connected with a space-time region O_1 are required to commute with all observables of another algebra which is associated with a space-like separated space-time region O_2. This axiom is also called Einstein causality.
(3) Translation covariance: if A is a net of local algebras of observables on an affine space, it is assumed that there exists a faithful and continuous representation $x \rightarrow \alpha x$ of the translation group in the group $AutA$ of automorphisms of A and $\alpha x(A(O)) = A(O + x)$, for any space-time region O and translation x.
(4) Spectrum condition: the support of the spectral measure of the operator associated with a translation is contained in the closed forward light-cone, for all translations. This ensures that negative energies cannot occur.

It is also important to define a couple of other notions that will be helpful to understand some ideas that will be discussed in the rest of the paper. First of all, we must define the notion of representation as a map that associates every element of an abstract C*-algebra $A(O)$ with the set of all bounded operators act-

[9] Since it is not possible to give a complete account of AQFT, I will give only a brief and sketchy presentation. The interested reader can see Halvorson and Mueger 2007 for a very technical and mathematical presentation, or Ruetsche 2012 for a bit more philosophical oriented analysis of the theory.

[10] According to a very general definition, a C*-algebra A is a complex algebra of continuous (bounded) linear operators defined on a complex Hilbert space, with the following important properties:

(i) A is (topologically) closed in the norm topology of operators;
(ii) A is closed under the operation of taking adjoints of operators.

[11] To complete our basic introduction to the structure of AQFT, we should specify that (i) local observables are defined as self-adjoint elements in local (non-commutative) algebras; and (ii) the state of a physical system is defined as a positive, linear and normalized function that associates elements of the local algebra of observables to real numbers.

[12] Here I follow the definition of the axioms as given by Rossanese (2016: 321).

ing on a Hilbert space *H*. A representation is said to be *irreducible* if the representation space *H* has no closed invariant subspaces. The notion of irreducibility is important because an irreducible representation is usually considered to represent an *elementary system*, which—ontologically speaking—is considered as fundamental. However, if we analyze the algebraic structure of AQFT, we realize that there are several possible (unitarily) inequivalent irreducible representations of the same algebra of observables generated by the canonical commutation relations (*CCRs*) for the field operators. Now, the Stone-von Neumann theorem proves that the algebra generated by the *CCRs* for the position and momentum operators has a representation in Hilbert space up to unitary equivalence. However, the Stone-von Neumann theorem is proved only for systems with a finite number of degrees of freedom. Since AQFT is a theory that describe physical systems with an infinite number of degrees of freedom, the Stone-von Neumann theorem does not hold in the context of AQFT. As we shall see, it is possible to "solve" this problem thanks to the superselection formalism.

We need also to define the notion of the Gelfand-Naimark-Segal-representation, that is, the GNS representation: "Let ω be a state on a C*-algebra $\boldsymbol{A}$. Then there exists a Hilbert space H_ω, a representation $\pi_\omega : A \rightarrow \boldsymbol{B}(H_\omega)$ of the algebra of observables, and a cyclic vector $|\xi_\omega> \in H_\omega$, such that for all $A \in \boldsymbol{A}$, the expectation values that the state ω assigns to the algebraic operator A is duplicated by the expectation value that the vector $|\xi_\omega>$ assigns to the Hilbert space operator $\pi(A)$. The *triple* $(H_\omega, \pi_\omega, |\xi_\omega>)$ is a cyclic representation because it contains a cyclic vector and it is called GNS-representation. It is unique up to unitarily equivalence. That is, if (H, π) is a representation of $\boldsymbol{A}$ containing a cyclic vector $|\psi>$ such that $\omega (A) = <\psi| A |\psi>$, then (H, π) and (H_ω, π_ω) are unitarily equivalent. A state ω on a C*- algebra $\boldsymbol{A}$ is pure if and only if its GNS-representation is irreducible; if its GNS-representation is reducible, the state is a mixed state".[13]

As said, the observables and their algebraic structure are at the center of this formalism, and hence AQFT can be interpreted thanks to the framework that we have described in the previous part of this paper. However, as mentioned, we have to deal with the problem of several different inequivalent irreducible representations of the algebras of observables in AQFT. In fact, if we need to identify the fundamental type-properties, we need to have a definite representation of the algebras of observables, or at least a definite representation for each quantum system described by AQFT.

The superselection formalism was originally proposed as a restriction on the nature and the scope of possible measurements. In the context of AQFT, a superselection rule is provided by Doplicher, Haag and Roberts (DHR) (1971 and 1974), who impose a superselection criterion according to which all the expectation values of all observables should uniformly approach the vacuum expectation values when the measurement region is far from the origin. Such a criterion allows DHR to find equivalent classes of irreducible inequivalent representations corresponding to charge superselection sectors. Thanks to the DHR analysis, it is then possible to consider as physical only the irreducible representations that are superselected by the DHR criterion. In other terms, the physical representations are the superselection sectors that can be reached from the vacu-

[13] Here I use the definition of the GNS-representation as given in Rossanese 2016: 322.

um sector through the action of local (unobservable) fields. If the space has the dimension of Minkowski space-time (or higher), those charge quantum numbers stand in a one to one correspondence to the labels attached to the irreducible inequivalent representations of the global gauge group. This means that it is possible to connect the algebraic structure to the symmetry structure of the theory. It is also possible to define a composition of charges in terms of a tensor product of those group representations, a conjugation of charges corresponding to the complex conjugate representations and a sign that is attached to each kind of charge, determining then if the system is fermionic or bosonic and the relevant statistics. In fact, quantum statistics arises from the structure of the category of representations of the observable algebra, which in particular is characterized by the fact that any GNS-representation is isomorphic to the category of localized transportable morphisms.

To sum up, DHR show that it is possible to recover all the properties of quantum fields from the analysis of superselection sectors. In particular, as already mentioned, they are able to recover the following physical structures: (i) properties of quantum number (baryon number, lepton number, and the magnitude of generalized isospin); (ii) composition law and conjugation of charge; (iii) exchange symmetry of identical charges—that is, statistics for quantum systems.

However, the DHR criterion cannot account for physical states with electric charge.[14] For this reason, Buchholz and Fredenhagen (BF) (1982) propose a different criterion in which the space-time region is replaced by an infinitely extended cone around some arbitrarily chosen space-like direction—and they then introduce the notion of *topological charge.* The fundamental idea is to consider almost local algebras and almost local operators in order to have an account of non-localizable charges. An almost local algebra is the set of all the elements which can be approximated by local observables in a diamond of radius r with an error decreasing in norm faster than any inverse power of r. It is important to notice that, in a theory formulated on a Minkowski space-time (such as AQFT), the results of the BF analysis are equivalent to those of the DHR analysis that we have briefly described above.

What is the moral of this quite technical part? I think that it is possible to accept Da Costa, Lombardi and Lastiri's ontology of properties also in the context of AQFT. In order to identify the type-properties that compose the bundle which represents a quantum system, we need to make three further steps with respect to their original proposal. First, we need to consider a different group of symmetry, namely the Poincaré group. Second, we need to take into account also a new kind of symmetry, namely the internal gauge symmetry. Finally, we have to deal with the inequivalent irreducible representations of the algebra of observables with the help of the superselection formalism. In particular, the internal symmetry considered in the analysis of the superselection formalism should always be represented by gauge symmetries. Quantum numbers manifest themselves with the existence of superselection rules for the states over the alge-

[14] This problem concerns the Gauss's law and to the fact that electric charge spreads space-like at infinity due to Coulomb's law. This means that electric charge cannot be properly represented by a localized operator such as those required in the definition of the DHR criterion.

bra of observables. The net structure of charge quantum numbers serves then to distinguish different species of physical systems and characterize their properties.

Now, the superselection formalism allows us to (super-)select all those representations of the algebras of observables that have a physical meaning. It then allows us to identify all the relevant observables and therefore all the type-properties of quantum systems. As in the context of the algebraic reformulation of QM, we can define states ρ as functionals over the algebras of observables in the following way: $\rho(A) = Tr <A\rho> = <A>_{\rho} \in R$, *for all* $A \in \boldsymbol{A}$. Da Costa, Lombardi and Lastiri (2013)'s proposal can be then generalized to the specific formalism of AQFT.

In the context of AQFT, it is therefore possible to identify all the relevant type-properties of a quantum system thanks to the algebraic structure of the theory and, in particular, thanks to the superselection formalism. Case-properties are then identified when one specific superselection sector is defined, that is, when we apply certain further conditions to the algebraic structure representing the quantum system in order to shift from a *universal kind of charge* to the *specific charge of that kind* we are interested in. Finally, we can have actualized case-properties when a measurement of that specific charge is performed—but, as we shall see, we first need a clear definition of what a measurement is in the context of AQFT.

It is also possible to recover the notion of particle. For example, Enss (1975) proposes an algebraic framework in order to give a definition of particles in terms of their local properties. As said, according to a first intuitive definition, a particle system is a stable physical system that cannot be decomposed into subsystems. Here "stable" means that the irreducibility of the system is conserved through any dynamical evolution. If we consider a particle as a collection of local observables, we have that the components of the one-particle system, as its properties (i.e., the observables), must "remain close" to each other. Moreover, as said, a particle should be localized (and in principle localizable) in a small region of space-time. In this sense, then, a particle could be understood as the neighborhood of a space-time point, that is, the localization point of a certain set of local observables. To put it in other terms, the notion of particle is defined by the set of particle states localized within a small region of space-time. However, there is a quantum phenomenon, the so-called *spreading*, that shows that the region where a particle is localizable increases with time. This entails also that it is impossible to use the notion of localization in order to distinguish one particle state from another, just because their respective localization regions can overlap after a certain time *t*. In order to solve this problem, Enss introduces a different notion of localization: a state can be "singly localized at time *t* with correlation-radius *r*". As such, a particle state "can be constructed by superposition of state-vectors which are localized at time *t* in various region of radius *r*" (Enss 1975: 36). There is also an operative way of constructing these states: "the singly localized states can be characterized by their inability to trigger a coincidence arrangement of two counters separated by a longer distance than *r*" (Enss 1975: 36). This means that any two one-particle systems would not overlap if two counters are enough separated; that is, if they are separated by a certain distance that is longer than *r*. In this case, "an N-fold localized state will trigger a coincidence arrangement of N-separated counters" (Enss 1975: 36).

It is then possible to consider Enss' construction as an example of a bundle theory of particles, where particles are defined as a collection of localized properties/observables (and hence states, namely particle states). This definition, however, is strictly dependent on the possibility to localize the properties of a particle, and then on the results of detection experiments. Enns proves some nice results in the context of Haag-Ruelle scattering theory, where it is possible to give an algebraically precise definition of the notion of detection (in terms of the notion of counter). Enns provides a clear formalism that links all the particle states to the measurement procedure, by showing how we can implement the process of particle detection in the algebraic framework. His definition allows also to define the notion of particle number in terms of the triggering of a set of widely separated counters—but only in the asymptotic limit (see also Rossanese 2021).

Conclusions

We have discussed an interesting proposal for an ontology of properties in the context of QTs. First of all, we have presented the original proposal of Da Costa, Lombardi and Lastiri and we have then tried to generalize such ontology in the context of QFT, and in particular in the context of AQFT. Of course, in this paper I have just drawn the first lines of a project that still shows the label "work in progress". In any case, in this conclusive section, I would like to stress some points on which the future work has to be directed.

We have defined a way to identify the relevant type-properties both in the context of QM and of QFT/AQFT. However, it is important to say something about the process of measurement, which allows us to measure the actualised case-properties, that is, the actual values of the properties that we have had identified thanks to choice of the preferred context and to the superselection formalism. Since it is important to provide a definite actualization dynamics, I think that it's worth mentioning the status of the measurement problem in the context QFT. Here I would take into account a schema for measurement that has been very recently proposed by Grimmer (2022).

Grimmer (2022) offers an interesting analysis of what he calls the pragmatic QFT measurement problem. He offers a case-by-case analysis of measurement frameworks for quantum fields, which are mainly based on the Unruh-DeWitt models. The basic idea is to define a measurement chain as a sequence of interactions, which carries the measured information from the quantum system to the record-keeping device. But, how to determine the end of the so-called measurement chain? It is possible to determine a pragmatic *Heisenberg-cut* such that the system is no longer quantum after the cut. For instance, we can consider that the decoherence of the system has occurred, or that the spontaneous wide-scale recoherence is "practically" impossible. He gives also three examples of possible cut: (i) from quantum fields to non fields systems ("field cut"), (ii) from relativistic systems to non-relativistic systems ("relativistic cut"), (iii) from factor III algebras of observables (the structure of the algebra in AQFT) to factor I algebras of observables ("split property cut").

Grimmer analyses also Fewster and Verch (2020)'s model of measurements as interactions in the specific context of AQFT. Fewster and Verch propose a measurement framework in terms of local interactions between AQFTs, where one acts as a local probe on the other. The localized dynamical coupling between the system and the probe gives rise to a scattering morphism, that is, the probe acts on the system as a scattering process, which depends on the probe observable measured and on the initial state of the probe. This measurement scheme allows to define the localized properties of the system in terms of the coupling regions between the system and the probe.

To conclude, Da Costa, Lombardi and Lastiri (2013)'s seems to be an interesting ontological option to interpret quantum theories and AQFT in particular. The fundamental ontology is then an ontology of type-properties identified by the space-time symmetries and by the gauge symmetries of the Hamitonian, together with the superselection formalism (for what concerns the specific case of AQFT).

It is possible to recover the notion of a particle as a bundle of localized properties (see Dieks and Lubberdink 2011 and Lombardi and Dieks 2016 for QM, and Rossanese 2021 for AQFT).

However, there is still work to do. On the one hand, it is first of all important to further develop Ardenghi, Castagnino and Lombardi (2009b)'s analysis to other gauge groups. On the other hand, the (pragmatical) measurement models of QFT should be further investigated, both in the context of standard formulation of QFT and in the context of AQFT.[15]

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[15] I would like to thank all the comments, suggestions and objections that I received at the 3rd Conference of the PRIN "The Manifest Image and the Scientific Image", "*Models, Structures and Representation*", where this paper has been originally presented. I would also like to thank two anonymous referees for the interesting comments, which have helped me to improve this paper. This work was supported by the Italian Ministry of Education, University and Research through the PRIN 2017 "The Manifest Image and the Scientific Image", prot. 2017ZNWW7F_004.

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