Empirical Success, Closeness to Evidence, and Approximation to the Truth

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Abstract

Realists and antirealists agree that different theories can be more or less empirically successful, even if they disagree on how to interpret this fact. Most of their arguments rely on how the notion of success is understood; still, few definitions of success are available, and their adequacy is doubtful. In this paper, we discuss some of these definitions and introduce a new measure of the success of a theory relative to a body of evidence aimed at overcoming some of their limitations. We moreover discuss how empirical success is connected to the approximate truth (or truthlikeness) of theories, a point of crucial importance for the defense of scientific realism.

Keywords: Empirical success, Evidence, Truthlikeness, Similarity, (Anti)realism.

1. Introduction

Today, the debate between scientific realists and antirealists is as lively and diverse as ever. A main point of contention is how to interpret the empirical success of our best theories: as a symptom of their approximate truth, as realists maintain, or instead as their ability to "save the phenomena", as antirealists suggest? One thing that both camps agree on, however, is the plain fact that theories can be, and often are, in fact, empirically successful, i.e., able to "account for" (fit, accommodate) a body of available evidence. It is moreover commonly assumed that in doing this, some theories may be better than others; in other words, that "empirical success" is a comparative notion, admitting of degrees.

In light of the above it is perhaps surprising that, as Malcolm Forster (2007: 589) notes, "[r]ealists and antirealists have not said much about how empirical success should be defined" (there are however important exceptions, discussed below). While much work has been devoted to defining adequate explications, e.g., of the empirical support or confirmation received by theories (Crupi 2021), or of their explanatory power relative to some evidence (Sprenger and Hartmann 2019: Ch. 7), it is not clear whether such notions are sufficient to exhaust that of success, and there are reasons to believe the contrary. In any case, defining success is not only interesting on its own, but also crucial to most of the arguments in the

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realism/antirealism debate. For instance, Laudan (1981) famously challenged scientific realists to explain in precise terms the link between the empirical success of a theory and its approximate truth or, to use his terminology, to justify the Downward Path (from approximate truth to empirical success) and the Upward Path (from empirical success to probable approximate truth). Of course, to accomplish such a task the realist needs an appropriate notion of success, one that can work together with the available explications of (probable) approximate truth (or truthlikeness, or verisimilitude). Antirealist equally cannot do without such a notion, at least if they want to be able to explain in what sense science develops and progresses toward increasingly successful theories (Niiniluoto 2019: Sect. 3).

This paper aims at systematizing some intuitions concerning various notions of empirical success found in the literature, in order to make explicit a couple of adequacy conditions that arguably should govern the use of the notion. In doing this, we hope to set some ground to further study, in a realist perspective, the links between success, on the one hand, and scientific progress as approximation to the truth, on the other. We start in Section 2 with a quick look at the current debate on realism and antirealism, emphasizing that both camps share the need for an adequate understanding of empirical success. In Section 3, we focus on three attempts to formally define such notion—due respectively to Hempel, Kuipers, and Zamora Bonilla—and point to some of their limitations. To overcome some of these, in Section 4 we introduce a new measure of success construed as the degree of similarity or closeness of a theory to the available evidence. Section 5 introduces the notion of the truthlikeness or verisimilitude of a theory and discusses how it interacts with empirical success as we propose to define it, in the light of Laudan's challenge. In Section 6 we offer some brief concluding remarks.

2. Why Success Matters

At the most general level, one can define scientific realism as "a positive epistemic attitude towards the content of our best theories and models, recommending belief in both observable and unobservable aspects of the world described by the sciences" (Chakravartty 2017). To be slightly more precise, realism is usually taken to be a package of views including (qualified versions of) three theses. The first is the metaphysical—or, depending on one's preferred parlance, ontological—thesis that the world that scientific theories aim to describe exists independently of our minds. The second is the semantic thesis that scientific theories, being attempts to describe the world and not just to systematize observations, make claims that must be taken literally, as having truth values. The third—the most interesting for our present purposes—is the epistemological thesis that our most successful sciences produce theories that offer approximately true descriptions of the aspects or fragments of the world that constitute their "targets".[1]

While the above package of theses is the backbone of any realist position, it seems fair to say that there are as many versions of scientific realism as there are scientific realists. It is not difficult to see why. Consider, for instance, an important issue raised by the realist's optimistic attitude towards our current best theories, from which embracing the epistemological thesis above follows naturally. Does such an attitude bring with it the commitment to the view that our best, most

[1] Such a characterization of realism follows quite closely the one proposed by Psillos (1999: XVIII) and is substantially equivalent to those suggested, among many others, by Niiniluoto (1999) and Chakravartty (2017).

successful theories get things basically right, and as a consequence, any mistake remaining in their descriptions of the world merely concerns matters of detail? Will the theories embraced by the scientists of the distant future be nothing but slightly amended versions of current most successful theories? According to Kyle Stanford (2015, 2021), a positive answer to such questions is what characterizes "classical" or "commonsense" realism, espoused in past decades by such authors as Smart (1968) and Putnam (1975).

Classical or commonsense realism, however, does not have much currency nowadays: in the majority of cases, Stanford points out, scientific realists today are more "historically sophisticated" than their predecessors, and therefore allow for the possibility of some future theoretical changes that will alter significantly the current scientific image of the world. As Stanford puts it, historically sophisticated realists take into proper account the revolutionary theoretical upheavals characterizing the past of science, and therefore tend to qualify their optimism by restricting it only to certain parts, or elements, of our current most successful theories. More specifically, so-called "selective realists" restrict their commitment to the parts or elements of our best theories that are responsible for their success, and that they maintain one can reliably identify (although different brands of selective realism differ concerning which parts of theories are deserving of realist commitment). Indeed, selective realism has become an important and lively tradition within the realist camp (see, e.g., Kitcher 1993, Psillos 1999, Cordero 2017, Alai 2021). But the qualified optimism of selective realists, as Stanford readily points out, is optimism nonetheless—that is, something that marks a difference between historically sophisticated realists, on the one hand, and antirealists, who do not embrace the epistemological thesis typical of realism, on the other hand.

One must mention, though, that full awareness of the track record of the scientific enterprise need not necessarily lead the realist to adopt a selective approach, or to be willing to concede that radical theory changes, analogous to the revolutionary ones that occurred in the past, will take place in the future. For instance, Fahrbach (2011, 2017, 2021) forcefully argues that our current best theories are of a different kind than past successful theories that have by now been discarded. In fact, our current best theories enjoy a much higher degree of success than theories of the past. On Fahrbach's account, this depends on both the quality and the quantity of the evidence supporting them, which is today enormously higher than it used to be in previous phases of the scientific development. In light of the extremely high level of support from the evidence enjoyed by our current best, most successful theories, Fahrbach suggests, the realist's embrace of the epistemological thesis is then by and large more justified today than it was in the past.

The above illustrates that the debate within the camp of scientific realism is today as lively and diverse as ever (see Alai 2017 and Saatsi 2018 for the state-of-the-art). More importantly for our present purposes, the preceding highlights that the notion of success—intuitively, the degree to which a theory or hypothesis H accounts for a body of evidence E—must play a key role within any viable realist position. Absent an appropriately defined notion of success, the epistemological thesis characterizing realism does not even make sense. In fact, the realist's optimism towards the content of our best theories and models hinges upon the fact that the realist views a theory's success as a (fallible) indicator of its (approximate) truth.

In "A Confutation of Convergent Realism" (1981: 32-36), Laudan famously challenged realists to show that there is in fact what he calls an "Upward Path", namely, that a theory's success provides one with appropriate epistemic warrant for the theory's (approximate) truth. Many current versions of scientific realism have arguably been developed, at least in part, precisely in order to meet the challenge posed in Laudan's paper—a task that, of course, can only be accomplished with an adequate notion of success in hand. However, it would of course be a mistake to think that success matters only to realists.

Antirealists, a no less diverse crowd than that of realists, also need an adequate notion of success, in the absence of which it is impossible to make sense of the development of science. Think of the idea that there is scientific progress—that our current best theories are in some relevant sense better than previous, by now discarded, theories. Laudan (1977) has offered an antirealist characterization of scientific progress in terms of the increasing problem-solving effectiveness of theories. In order for such an account to work, a notion of success in problem-solving effectiveness is required. And even Kuhn, who viewed the development of science not as that of an enterprise getting nearer and nearer to "some goal set by nature in advance", but rather, "in terms of evolution from the community's state of knowledge at any given time" (1962/1970: 171), needed the notion of success to account for the theory-choices made by scientific communities.[2] To mention but one more instance of how antirealists too need an appropriate notion of success in order for their accounts of science to work, recall in what terms van Fraassen defines the aim of inquiry pursued by the constructive empiricist. Such an aim is empirical adequacy, where an empirically adequate theory is one that "saves the phenomena" in the sense that "what it says about the observable things and events in this world […] is true" (1980: 12). Of course, different theories may be more or less adequate in van Fraassen's sense, meaning that they will save a larger or smaller part of the phenomena; again, the notion of success in the sense in which we deal with it here is obviously involved.

In sum, the notion of success plays a central role in any viable account of science, be it realist or antirealist. Importantly, realists and antirealists need not disagree on the best way to characterize and measure success. To be sure, realists read into success something—as mentioned, a (fallible) indication of (approximate) truth—that antirealists maintain cannot be read into it. Still, both realists and antirealists agree, for instance, that success is a matter of degree: we need a comparative notion, since we want to be able to meaningfully say that a certain theory is more (less) successful than another (or as successful as another). Moreover, even the most optimist of realists readily agrees with antirealists that for most of the time scientists deal with theories that have already been falsified, and yet enjoy a certain degree of success, and must therefore be taken seriously nonetheless.

In the next two sections, we briefly review some selected attempts to provide rigorous explications of the notion of empirical success in the form of a measure of the degree of the success enjoyed by a hypothesis or theory with respect to some available body of evidence. Taking our cue from such attempts, we also present a new measure of success satisfying several adequacy conditions which arguably govern intuitive assessments of the relative success of different hypotheses.

[2] See Shan 2019 for a recent attempt to revive the accounts of progress put forward by Laudan and Kuhn.

3. Measuring Success

The effort of clarifying the links between success and approximate truth has led several scholars to develop formal accounts of both notions. These accounts provide rigorous definitions of the success enjoyed by a theory or hypothesis *H* relative to a body of evidence *E*, sometimes in the form of measures of such success (cf. Niiniluoto and Tuomela 1979: Ch. 7; Sprenger and Hartmann 2019: Ch. 7). Note that such accounts assume (in line with much discussion within general and formal philosophy of science) that it is possible to talk of the success of *H* with respect to *E* in a sufficiently general and abstract sense, i.e., not only relative to specific examples, scenarios or contexts of application.

A classic example of such an approach is Hempel's discussion of the notion of the "systematic power" of *H* with respect to a body of evidence or information *E*, first introduced in the last part of his celebrated 1948 paper on the logic of explanation (co-authored with Paul Oppenheim, reprinted in Hempel 1965). In Hempel's intentions, systematic power includes both the predictive and the explanatory performance of *H*, in agreement with the well-known thesis about the symmetry between explanation and prediction (Hempel 1965: 279). The intuition is that *H* has great systematic power when *H* "covers" a great part of the evidence *E*, in the sense that *H* entails a high proportion of the content of *E*. To make this precise, Hempel introduces an (epistemic) probability distribution *p* for the relevant language in which *H* and *E* are expressed (*p* is defined on the possible worlds that can be described by such language or, in Hempel's Carnapian jargon, on its constituents or state-descriptions). He then defines the *content* of a proposition *X* as the measure $cont(X) = 1 - p(X)$, in agreement with the intuition (shared by both Popper and Carnap, among others) that the greater the information content of *X*, the smaller its probability, i.e., the "size" of the set of possible worlds compatible with *X*. Finally, Hempel (1965: 287) defines the systematic power of *H* with respect to *E* as the ratio of the common content of *H* and *E* to the content of *E*:

1) $syst(H,E) = \frac{cont(H \vee E)}{cont(E)} = p(\neg H|\neg E)$

While Hempel introduces *syst*(*H*,*E*) as a measure of explanatory and predictive power, it is quite clear that it can be employed as a measure of the success of *H* on *E*; for instance, as noted by Ilkka Niiniluoto, *syst* can be used to formally explicate Laudan's notion of problem-solving effectiveness (Niiniluoto 1990: 438-39). Indeed, *syst* seems to capture well some intuitively sound conditions characterizing the notion of empirical success. As an example, if *H* deductively entails *E*, and in this sense it is maximally successful, then $p(H|\neg E) = 0$ and hence $syst(H,E) = 1 - p(H|\neg E)$ receives its maximum value 1, as expected. If *H* is tautological, it has no information content and, as such, it tells nothing about *E* since it entails no contingent consequences; accordingly, since $p(H|\neg E) = 1$, then *syst*(*H*,*E*) is minimal, i.e., 0.

An interesting consequence of Hempel's definition is that it allows us to compare falsified theories as far as their relative success is concerned. As we shall see in the following, this can be defended as an adequacy condition for any satisfactory explication of empirical success (Kuipers 2000: 94). If *H1* and *H2* are falsified by *E*, in the sense they both entail ¬*E*, they can still have different degrees of success. A more surprising consequence is that, as one can check, if *H1*, but not *H2*, is falsified by *E*, it could be that *syst*(*H1*,*E*) is greater than *syst*(*H2*,*E*). This is the case, for instance, when *H1* is a highly informative but falsified theory, whereas *H2* is compatible with *E* but uninformative: in the extreme case where *H* is tautological, its systematic power is 0, i.e., the minimum.[3]

A less welcome consequence of Hempel's definition 1 above is however the following: if *H1* entails *H2*, then *H1* is always at least as successful as *H2*. Intuitively, this is because when *H1* is logically stronger than *H2*, it entails at least all the content of *H2* and perhaps more: accordingly, it cannot be less successful than *H2*. Formally, the reason is simply that if *H1* entails *H2*, then $p(H1|\neg E) < p(H2|\neg E)$ and hence $p(\neg H1|\neg E) > p(\neg H2|\neg E)$, which means that *syst* is always greater for *H1* than for *H2*. This result is troubling not only if empirical success is conceived as an indicator of (approximate) truth, but also if it is construed, more generally, as a cognitive utility guiding theory-choice, or it is used to define scientific progress, as Laudan suggests (cf. Niiniluoto 1990: 443). In fact, it implies that, if *H* enjoys some success relative to *E*, it is sufficient to add to *H* some piece of information *X* not already entailed by it to obtain a new theory *H* & *X* which is no less successful than *H*, even if the added information *X* is completely irrelevant or even false relative to *E*. In other words, increasing the empirical success of *H* becomes a "child's play": just strengthen *H* by conjoining it with any proposition *X* whatsoever (like "the Moon is made of green cheese").[4] In the extreme case, *X* can even be ¬*H*: in fact, success is maximized by an inconsistent, and hence maximally informative, theory.

The principle according to which success should co-vary with logical strength is highly problematic and, we argue, should be rejected as an adequacy condition for a measure of empirical success. However, it follows from Hempel's purely probabilistic account of the notion. Partially motivated by this problem, some have developed non-probabilistic explications of success. We shall briefly discuss two such accounts.

The first is due to Theo Kuipers who, in a series of works (Kuipers 1987, 2000, 2019), has defended a form of "constructive realism" based on a sophisticated analysis of the relationships between theories, evidence, and truth within a broadly "structuralist" framework. In doing so, he defines and discusses many different notions (like confirmation, progress, and truthlikeness) including that of empirical success. Cutting Kuipers' account to the bones, he distinguishes (following Laudan and others) between the "problems" and the "successes" of theories. Problems of *H* are established anomalies or counter-examples to *H*; successes of *H* are established facts that can be derived from it. Of course, *H* is the more successful, the more successes and the less problems it has. However, Kuipers is careful to emphasize that even if strictly speaking any counter-example to *H* falsifies it, this is not a sufficient reason to plainly reject *H* or consider it necessarily worse (in terms of empirical success) than a non-falsified theory. In his own

[3] In recent years, much work has been devoted to the logic of explanatory power, partly inspired by Hempel's early efforts (for the state-of-the-art, see Sprenger and Hartmann 2019: Ch. 7). Different probabilistic measures of the explanatory power of *H* with respect to *E* have been proposed, and interesting results about their axiomatic characterization and reciprocal relations obtained. Interestingly, none of these measures satisfies the requirement just discussed: if *E* falsifies *H*, the latter's degree of explanatory power is either undefined or minimal. This suggests that the notion of empirical success is richer than, even if connected to, that of explanatory power.

[4] This "child's play" objection is also standard in the truthlikeness literature, where it was raised against some earlier definitions of such notion (see, e.g., Kuipers 2000: 254).

words, theory evaluation has to "take falsified theories seriously" (Kuipers 2000: 94). This is reflected in his basic definition of comparative success (Kuipers 2000: 112, notation modified):

2) Theory *H1* is more successful than theory *H2* iff i) the set of problems of *H1* is a subset of that of *H2*; and ii) the set of successes of *H2* is a subset of that of *H1*; and iii) in at least one case the relevant subset is a proper subset.

In other words, if *H1* has at least one more success besides those of *H2*, or *H1* has at least one less problem than those of *H2*, *H1* is more successful than *H2*, and the shift from *H1* to *H2* counts as an instance of progress, understood as increasing success. As Kuipers notes, the assessment of the relative success of two (or more) different theories is always relative to a body of empirical evidence available at some point in time. Consequently, new evidence may always change the comparative judgment in the above definition.

It is worth noting that Kuipers' definition, just like Hempel's, allows one to compare falsified theories with respect to their relative success. If *H2* is falsified (i.e., its set of problems is not empty), *H1* may improve on it, for instance, by retaining all its problems and successes, and adding some more successes. In such a case, *H1* and *H2* are both falsified, but *H1* is more successful than *H2*. However, if *H1* is falsified and *H2* is not, *H1* cannot be more successful than *H2*, since in such a case, even if the set of successes of *H1* can properly include that of *H2*, the set of problems of *H1* cannot be a subset of that of *H2* (since the latter is empty and the former is not). Thus, Kuipers' basic definition does not satisfy the condition that falsified theories may be better than non-falsified ones, which is instead respected by Hempel's measure. On the other hand, if *H1* entails *H2*, then *H1* has all the problems and the success of *H2*; however, it could have no more successes and strictly more problems than *H1*, and so be less successful than it. Thus, Kuipers' definition satisfies the condition that empirical success does not necessarily co-vary with logical strength, a condition that Hempel's measure instead fails to meet. As we shall see in the next section, it is possible to define a notion of success very similar to Kuipers' one but eschewing the limits of both Kuipers' and Hempel's approaches.

Before turning to this, let us briefly discuss an account due to Jesús Zamora Bonilla (1992, 1996, 2000), providing another important step toward our own approach. Following Kuipers, Zamora Bonilla adopts a structuralist approach to theory representation. For our purposes, it is sufficient to focus on the simplest measure he discusses, which exhibits some interesting features. Zamora Bonilla introduces his measure as a measure of the estimated truthlikeness of a theory given the available evidence; as we suggest, it is more properly construed as a measure of "evidential similarity", i.e., as a measure of success defined as closeness to the empirically established truth (cf. Zamora Bonilla 1992: 347-49). The measure is defined as the product of the similarity $s(H,E)$ of theory H to evidence E and of the "rigor" $r(E)$ of the evidence, as follows (Zamora Bonilla 1996: 29; notation modified):

3) $evsim(H,E) \equiv s(H,E) \times r(E) \equiv \frac{p(H\&E)}{p(H \vee E)} \times \frac{1}{p(E)} = \frac{p(H|E)}{p(H \vee E)}$

Here, $s(H,E)$ measures, in probabilistic terms, the "overlap" between H and E; $r(E)$ is just the reciprocal of the probability of the evidence, taken as a measure of its informativeness. Measure *evsim* takes its maximum value if H entails E, i.e., when it is maximally successful, and has a number of other interesting features

(Zamora Bonilla 1996: 31ff). For our purposes, the main limitation of this account (that Zamora Bonilla carefully discusses in section 3 of his paper) is that it does not allow one to assess the relative success of falsified theories: in fact, if E falsifies H, then $evsim(H,E)$ is always 0, the minimal possible degree of success. On the other hand, we believe that Zamora Bonilla's account captures a crucial aspect of the notion of empirical success, i.e., that it should measure how "close" a theory is to the available evidence; his probabilistic measure $s(H,E)$, however, is too crude for this purpose. In the next section, we build upon this basic intuition in order to develop a more adequate notion of empirical success as similarity to evidence.

4. Success as Similarity to Evidence

The empirical success of H should depend on how well H accounts for the available evidence E. One natural way to spell out this intuition is defining the success of H on E in terms of the content of E which is also conveyed by H. As we saw in the previous section, Hempel's measure of systematic power does exactly this by employing a purely probabilistic notion of content (and hence of success), but it has some conceptual shortcomings. To avoid these, we suggest here another way of defining success, partially inspired by the proposals by Zamora Bonilla and Kuipers discussed above.[5]

The central idea is that H is the more successful the closer it is (in a suitably defined sense) to evidence E. To keep things simple, we rely on a quite minimal framework.[6] We assume that the evidence E is a collection of individual facts, each described by a single "basic proposition" of a finite propositional language. By "basic proposition" we mean an atomic proposition or its negation (in other words, basic propositions do not contain connectives except, possibly, for the negation). E can then be represented either as a set of m basic propositions or as their conjunction, the latter being the strongest evidential statement acceptable at a given moment in time. So, if A, B, C are atomic propositions, E could be expressed, for instance, both as $\{A, \neg B, C\}$ or as $A \,\&\, \neg B \,\&\, C$. Similarly, a theory or hypothesis H is a (consistent) collection or conjunction of k basic propositions of the same language. (Alternatively, one can think of such a collection as set of empirical consequences of a more complex theory at the observational level.)

The following terminology seems quite natural. Suppose that B is a basic proposition which appears as an element or conjunct of E. Then, we shall say that B is a (empirical) "match" of H if H entails B; that B is a (empirical) "mistake" of H if H entails the negation of B; and that B is a (empirical) "lacuna" of H if H does not entail B nor its negation. (Note that a lacuna, in this sense, is not an element or conjoint of H, but, so to speak, a "gap" of H with respect to E.) Intuitively, the matches of H count in favor of its empirical success; the mistakes and lacunae of

[5] A *caveat* may be relevant at this point. Following much of the literature, in this paper we leave on a side one important problem concerning success, i.e., the distinction between accommodation and prediction (which is crucial, e.g., in statistics, where success is defined as fit to the data). In other words, we are separating the problem of defining the success of a theory in terms of its matches (and mistakes) and that of defining when such matches are "genuine" or "fudged" (as with overfitting in statistics). The latter problem is carefully discussed by Forster (2007); for a very recent discussion of "predictivism", see Crupi 2023.
[6] The present framework is borrowed from the so-called basic feature approach to truthlikeness (Cevolani et al. 2011; Cevolani et al. 2013; Cevolani and Festa 2021) to be discussed in the next subsection.

H detract from it. More formally, let us denote with t_E (for "true with respect to *E*") and f_E (for "false with respect to *E*"), respectively, the number of empirical matches and mistakes of *H*. Then, we can define the following simple measure of the empirical success of *H* with respect to *E*:

4) $es(H,E) = \frac{t_E}{m} - \frac{f_E}{m}$

Recalling that m is the number of the evidential statements in *E*, *es*(*H*,*E*) amounts to the normalized difference between the number of matches and mistakes of *H*. Note that, even if the lacunae of *H* are not explicitly mentioned, they count against the empirical success by lowering *es*(*H*,*E*): if *H* has many lacunae, it cannot be much successful according to such measure. In the extreme case, when *H* entails no empirical consequence at all (i.e., it is an empty set or conjunction, with $k = 0$), it is completely "lacunose" (so to speak), and its degree of success is 0. In such case, with a slight abuse of language, we shall say that *H* is tautological, meaning that it entails no basic propositions at all.

As we argue, our simple definition satisfies several intuitive desiderata on the notion of empirical success. For instance, if *H* is "maximally successful" in the sense that it entails *E* (and hence *H* entails all the m conjuncts of *E*), then its degree of success is $es(H,E) = \frac{m}{m} = 1$, which is the maximum possible. On the other hand, if *H* has at least one mistake or one lacuna, then *es*(*H*,*E*) will be lower than 1. The minimal degree of success (i.e., –1) is reached when theory *H* entails the negations of all the m conjuncts of *E*, i.e., *H* is maximally unsuccessful; if *H* only makes mistakes, then its degree of success is always negative. In this connection, the degree of success of a tautology (in the sense defined above) is a sort of natural middle point: from a qualitative point of view, we could say that *H* is "successful" if *es*(*H*,*E*) > 0, "unsuccessful" if *es*(*H*,*E*) < 0, and "empirically neutral" otherwise. Note that, according to this simple measure, a non-tautological theory *H* counting exactly the same number of matches and mistakes has the same degree of success as a tautological one, i.e., 0.

It is also easy to check that our measure satisfies all the conditions discussed in the preceding section, thus allowing for simple assessments of relative success of different theories. In particular, it conveys as special cases Kuipers' comparative assessments of success: if *H1* has more matches and no more mistakes, or less mistakes and no more matches, than *H2*, then *H1* is more successful than *H2*. Moreover, *es* avoids the unwelcome consequence of Hempel's probabilistic measure. If *H1* entails *H2*, this does not imply that *H1* is more successful than *H2*. To see this, suppose that *H2* has only matches, and *H1* adds to these some mistakes: then *H1* entails *H2* but *H1* will be less successful than *H2*.

To sum up, we list below a number of conditions governing the notion of empirical success, which are satisfied by our measure. Without attempting here a detailed defense of all of these conditions, we suggest that they may work as adequacy conditions for any viable explication of success, or at least that they should be take into account when discussing one. Note that we do not claim originality concerning such conditions, partly borrowed from extant literature, and note also that we are not implying that they need to be logically independent or exhaustive. Assuming that *E* represents all the available evidence with respect to which two theories *H1* and *H2* are evaluated in terms of their relative success, we have:

ES1. If both *H1* and *H2* entail *E*, they are equally successful.

ES2. If *H1* entails *E*, and *H2* does not entail *E*, *H1* is more successful than *H2*.

ES3. If both *H1* and *H2* are falsified by *E*, they are not necessarily equally successful.

ES4. If *H1* is falsified by *E*, and *H2* is not falsified, *H1* may be more successful than *H2*.

ES5. If *H1* entails *H2*, *H1* may be more, equally, or less successful than *H2*.

ES6. If *E* entails both *H1* and *H2*, and *H1* entails *H2*, *H1* is at least as successful as *H2*.

ES7. If *H* entails *E*, and *E* entails *E'*, *H* is more successful on *E* than on *E'*.

Some comments are in order. The first two conditions deal with non-falsified theories, i.e., theories which are compatible with the evidence *E*. ES1 says, in a sense, that the best a theory can do is to fully entail the evidence: among such "maximally successful" theories, there is no difference as far as success is concerned.[7] ES2 says that maximally successful theories are more successful than theories that have mistakes or lacunae. The next two conditions concern instead theories that are falsified by the evidence. ES3 is the basic requirement that falsified theories are not all on the same level: it is possible to compare them according to their relative success. ES4 specifies that falsified theories may be even more successful than non-falsified ones (as discussed above in relation to Hempel's proposal). The next couple of conditions govern the relationships between success and logical strength. ES5 emphasizes that there is no general link: logically stronger theories may be more or less successful than weaker ones, depending on how they relate to *E*. However, in the rather special (and unrealistic) case where two theories are both verified by *E* (there are no mistakes, but only matches, for both *H1* and *H2*), the logically stronger is also the more successful. Finally, ES7 concerns the success of a single theory *H* with respect to two pieces of evidence: if *H* is fully successful on both of them, its degree of success will be higher on the more informative piece of evidence.

A full discussion and defense of ES1-ES7 will have to be left for another occasion. In what follows, we focus instead on some interesting methodological consequences of our definition of success. Before doing this, however, let us note a further, final point. A theory *H* can "go beyond the evidence" in the sense that it entails more (or different) empirical consequences than those that, collectively taken, form *E* (this happens for sure if *k* is greater than *m*). This implies that estimates of the success of *H* are always relative to the available body of evidence *E* and always revisable: if at a later time one discovers that some *B* (not already contained in *E*) is true (and hence becomes part of *E*), the empirical success of *H* relative to the new evidence may increase (if *B* is a match of *H*), decrease (if *B* is a mistake of *H*) or remain the same (if *B* is a lacuna of *H*).

5. From Success to (Expected) Truthlikeness, and Back

Our main reason for dealing with measures of empirical success like *es* is, as mentioned, to study the relationship between success, on the one hand, and truthlikeness, on the other hand, from a realist's point of view. Let us emphasize, however, that our *es* measure should be of interest also to the anti-realist. Indeed, anti-real-

[7] Of course, maximally successful theories may well differ under other, important respects, like their simplicity, unification power, etc.

ists need a way of comparing theories with respect to their relative empirical success, unless they are prepared to reduce all kinds of theory assessment to a binary, all-or-nothing classification of "successful" vs. "unsuccessful" theories (cf. Kuipers 2000: 94). Since all theories in the history of science (or at least those accepted or taken seriously for some time) probably had some degree of success, one can argue that the anti-realist needs at least a comparative notion of success obeying conditions ES1-ES7 above. Our measure *es* provides such a notion and, we suggest, is perfectly acceptable to anti-realists, since it does not involve any reference to truth beyond the evidence.

Having clarified this point, let us now turn to the idea of truth approximation. In a nutshell, a truthlike theory is one that provides much true information, and few false information, about its target domain. If a theory H is highly successful with respect to the available evidence, the realist feels confident that H is on the right track toward the truth. To put it differently, from the realist's point of view, the success of theories is a fallible, empirical indicator of their actual closeness to the (unknown) truth, and speaking of assessments of the relative truthlikeness of different theories is fully meaningful. To clarify these intuitions, however, the notion of truthlikeness needs to be defined in more details.

Interestingly, the same approach we adopted to define success can be used here to define (expected) truthlikeness (Cevolani et al. 2011; Cevolani et al. 2013; Cevolani and Festa 2021). Given a finite propositional language with n atomic propositions, the strongest true statement of such language will represent "the whole truth" about the target domain. (Of course, we assume that n is not smaller than either m and k.) This statement T is the conjunction of the n true basic propositions of the language. Intuitively, T is the most complete true description of the actual world, given the resources of our language; the other "constituents" of the language (conjunctions of n basic propositions) describe all the other possible worlds which are not actual (in total, there are 2^n constituents or possible worlds, including T). A theory H will be the more truthlike or verisimilar, the closer or more similar H is to T. In general, given a theory H and a constituent W, the similarity of H to W will be measured as:

5) $sim(H, W) = \frac{t_W}{n} - \frac{f_W}{n}$

i.e., as the normalized difference between the number of matches and mistakes of H with respect to W. Accordingly, the truthlikeness or verisimilitude of H is defined as:

6) $vs(H) = sim(H, T) = \frac{t_T}{n} - \frac{f_T}{n} = \frac{t}{n} - \frac{f}{n}$

where we can avoid the subscript "T" since here matches and mistakes are properly true and false, respectively. Note that the truthlikeness of H is maximal (and equal to 1) when H is the truth T itself; it is minimal (and equal to –1) when H is the conjunction of the negations of all basic truths. A tautology has 0 truthlikeness; a non-tautological theory is more or less verisimilar than it, depending on the balance of basic truths and falsehoods it entails: the more truths and the less falsehoods, the better in terms of closeness to the truth.[8]

[8] In this connection, one should note that we are employing here the simplest possible measure of truthlikeness proposed by Cevolani et al. 2011. In their more general account, the relative "weight" of truths and falsehoods in assessing the verisimilitude of H can be

Note that the truthlikeness *vs*(*H*) of *H* is well-defined only assuming that the whole truth *T* is actually known. Of course, this is not what happens in all interesting cases of scientific inquiry. Typically, an inquirer can at best rely on a body of evidence *E*, assumed to be true, and try to assess the estimated truthlikeness of different theories on the basis of such evidence. Such "educated guesses" about estimated truthlikeness are, for the realist, the best one can do by construing empirical success as a fallible indicator of the theory's "real" truthlikeness. To make this idea clear, we can follow Niiniluoto (1987, 2017) in defining estimated truthlikeness as the expected value of the actual truthlikeness of a theory. Assuming that a (epistemic) probability distribution *p* is defined on the possible worlds (constituents) of our language, we define the expected truthlikeness of *H* on *E* as follows:

7) $evs(H|E) = \sum_{W_i} sim(H, W_i) \times p(W_i|E)$

i.e., as the sum of the degrees of truthlikeness of *H* in each possible world W_i, weighted by its corresponding probability given the evidence *E*. In words, *evs*(*H*|*E*) is high when *H* is very close to (highly verisimilar in) the possible worlds that the evidence indicates as highly probable. Assuming that the evidence is veridical, *evs*(*H*|*E*) is a fallible estimation of *H*'s actual truthlikeness, that can be revised as new evidence becomes available. Note that, as evidence increases, such estimate becomes increasingly reliable; in the limit, when *E* singles out just one possible world (the actual one, described by *T*), the expected truthlikeness of *H* is the same as its actual truthlikeness.

Equipped with defensible explications of the notions of empirical success and (expected) truthlikeness—in the form of the measures *es*, *vs*, and *evs*—we can now deal with some issues of central importance in the debate between realists and antirealists. In particular, we can re-formulate Laudan's Downward and Upward Paths as follows (cf. Niiniluoto 1999: sections 6.4-6.5):

DP) If *H* is (highly) verisimilar, it is (highly) successful.

UP) If *H* is (highly) successful, it is expected to be (highly) verisimilar (its degree of expected truthlikeness is high).

These two principles provide a bi-directional link between the success of *H* and its (expected) truthlikeness (or probable approximate truth, in Laudan's jargon): if *H* is actually verisimilar (something we cannot ascertain), it should enjoy a high degree of empirical success; vice versa, if *H* is highly successful on *E*, then its degree of expected truthlikeness on *E* should be comparatively high. Laudan maintains that realists should accept in general both DP and UP, and should provide good arguments in their support. However, there are good reasons to think that these principles are too strong, and therefore realists need not commit to them (cf. Niiniluoto 1999, 2017, 2019). Indeed, one can show that both DP and UP are violated if the measures *es*, *vs*, and *evs* discussed in this paper are employed as adequate explications of the relevant notions.

In fact, one can prove that none of the following two principles (which are nothing but the 'translation' of Laudan's in our present framework) holds in general:

different, so that, for instance, the "loss" in verisimilitude due a mistake is greater than the "gain" due to a match. Here, for the sake of simplicity, we are instead assuming that matches and mistakes are equally weighted in assessing truthlikeness.

DP') If $vs(H)$ is high, then $es(H,E)$ is high.

UP') If $es(H,E)$ is high, then $evs(H|E)$ is high.

The main reason why these principles fail in general is very simple: as stated, they are completely silent on what E is, more specifically, on the quality of the evidence upon which the relevant assessments of success and expected truthlikeness are performed. As we saw in Section 4, however, the precise relationship between H and E (here construed as the closeness of H to E) is obviously crucial to assess the success of H on E. In other words, the information provided by E must play a crucial role in evaluating the links between success and (expected) truthlikeness—a role that DP' and UP' ignore altogether.

Two simple (if rather abstract) counterexamples will be sufficient to show why the two principles are untenable in general. Suppose first that H is highly verisimilar, meaning that H is very close to T, i.e., H has many matches and very few (or none) mistakes. (Of course, this is something that one cannot ascertain, and that we assume for the sake of the argument). Moreover, suppose that E is very uninformative, i.e., it entails very few evidential statements. It follows that the success of H on E could be very low, for the simple reason that H and E could well have very few elements in common, or even none if either H and E are "disjoint" or E is tautological. In other words, even if $vs(H)$ is high (as assumed), $es(H,E)$ can be very low (or even 0 in the extreme cases mentioned). This shows why DP' cannot hold in general. (To be sure, it can happen that H is highly verisimilar, E is uninformative in the sense just defined, and still H is highly successful on E, because it entails all the few elements of E; this, however, doesn't need to happen in general.)

As for UP', a similar counterargument can be given. Suppose, as before, that E is very uninformative, for instance because E consists just of one evidential statement B. Moreover, suppose that H not only entails such B (and hence it is maximally successful) but, as an extreme case, it is equivalent to B (and hence to E). In such case, $es(H,E)$ is maximal, but $evs(H|E)$ may be very low, especially if n is very high: in fact, H will be very uninformative, and hence cannot have a high degree of expected truthlikeness. In other words, UP' cannot hold in general.

The lesson to be drawn from the preceding discussion is that the evaluation of methodological principles like DP and UP cannot be made in general, but only on a case-by-case basis, by taking into account the specific body of evidence E available in the relevant context. This, however, does not mean that nothing can be said concerning the relations between success and truthlikeness. Indeed, a reformulation of principles DP and UP suggests itself as a possible way out of the counterexamples just discussed:

DP") If E is (highly) informative and H is (highly) verisimilar, then H is (highly) successful on E.

UP") If E is (highly) informative and H is (highly) successful on E, then H is expected to be (highly) verisimilar on E (its degree of expected truthlikeness on E is high).

These new conditions make clear the role of the evidence E presently available in assessing the link between the success of some theory H on that evidence and its (expected) truthlikeness. Of course, a rigorous formulation of DP" and UP" would require a formal explication of the "informativeness" of E, possibly in the form of some measure similar to the ones already discussed. This would allow one to precisely formulate new principles—comparable to DP' and UP' above—and possibly to prove the existence of lower and upper bounds on the informativeness of *E* in relation to the success and (expected) truthlikeness of *H*. In this connection, we suggest that the framework presented here may be instrumental in proving that DP" and UP" actually hold under suitably defined conditions, a task that we have however to leave for the future.

6. Concluding Remarks

We started the paper by reviewing some aspects of the current debate between realists and antirealists, focusing in particular on the notion of empirical success of a theory or hypothesis *H*. That the relative success of different theories is a crucial ingredient of their evaluation and comparison is probably one of the few undisputed claims in such debate. In order to better assess the competing claims of realists and antirealists—and in particular the realist tenet that success is a fallible indicator of truthlikeness or approximate truth—we considered some formal explications of the notion of success, advanced by Hempel, Kuipers, and Zamora Bonilla. This led us to put forward a new definition of success (in the form of a measure defined on propositional languages) that, we argued, satisfies several adequacy conditions governing such notion, while overcoming the limitations of previous measures. In a nutshell, our definition construes the success of *H* as its similarity or closeness to the available body of evidence *E*.

In the final part of the paper, we showed how our account allows one to rigorously tackle some crucial aspects of the debate, and especially the discussion of the relationships between empirical success and (expected) truthlikeness. In this connection, our conclusions have been partly negative: one cannot prove, in general, strong "success theorems" (in the sense of Kuipers 1987, 2019) guaranteeing that high verisimilitude implies high success, or, vice versa, that high success implies high expected verisimilitude. In that sense, there is no general answer, on the part of the realist, to Laudan's challenge concerning the Upward and Downward Paths.

On a more positive note, we argued that Laudan's principles DP and UP are too strong, and therefore there is no reason for realists to embrace them without proper qualifications. This is because such principles ignore the issue of the quality of the available evidence, which becomes instead apparent in our account.

Moreover, such account has a number of advantages with respect to other proposals. First, it provides a defensible notion of success, satisfying a number of adequacy conditions discussed in the literature, but violated, at least in part, by other explications of success. Second, such notion is useful and perfectly acceptable also by the antirealist, thus providing a common ground for further discussion. Finally, our account provides a unified treatment of success (as closeness to observational truth) and of truthlikeness (as closeness to the whole truth) suggesting limited, but relevant, success theorems governing their relations. In this respect, further work is needed to explore both the potential and the limitations of the approach defended here.[9]

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References

Agazzi, E. 2017 (ed.), *Varieties of Scientific Realism: Objectivity and Truth in Science*, Cham: Springer.

Alai, M. 2017 "The Debates on Scientific Realism Today: Knowledge and Objectivity in Science", in Agazzi 2017: 19-47.

Alai, M. 2021, "The Historical Challenge to Realism and Essential Deployment", in Lyons, T.D. & Vickers, P. (eds.), *Contemporary Scientific Realism: The Challenge from the History of Science*, Oxford: Oxford University Press, 183-215.

Cevolani, G., Crupi, V., & Festa, R. 2011, "Verisimilitude and Belief Change for Conjunctive Theories", *Erkenntnis*, 75, 183-202.

Cevolani, G. & Festa, R. 2021, "Approaching Deterministic and Probabilistic Truth: A Unified Account", *Synthese*, 199, 11465-489.

Cevolani, G., Festa, R., & Kuipers, T.A.F. 2013, "Verisimilitude and Belief Change for Nomic Conjunctive Theories", *Synthese*, 190, 3307-24.

Chakravartty, A. 2017, "Scientific Realism", in Zalta, E. (ed.), *The Stanford Encyclopedia of Philosophy*, Summer 2017 Edition.

Cordero, A. 2017, "Retention, Truth-Content and Selective Realism", in Agazzi 2017: 245-56.

Crupi, V. 2021, "Confirmation", in Zalta, E. (ed.), *The Stanford Encyclopedia of Philosophy*, Spring 2021 Edition.

Crupi, V. 2023, "The Case of Early Copernicanism: Epistemic Luck vs. Predictivist Vindication", preprint available at http://philsci-archive.pitt.edu/id/eprint/22583 (accessed 2023-11-03).

Fahrbach, L. 2011, "How the Growth of Science Ends Theory Change", *Synthese*, 180, 139-55.

Fahrbach, L. 2017, "Scientific Revolutions and the Explosion of Scientific Evidence", *Synthese*, 194, 5039-72.

Fahrbach, L. 2021, "We Think, They Thought: A Critique of the Pessimistic Meta-Meta-Induction", in Lyons, T.D. & Vickers, P. (eds.), *Contemporary Scientific Realism: The Challenge from the History of Science*, Oxford: Oxford University Press, 284-311.

Forster, M.R. 2007, "A Philosopher's Guide to Empirical Success", *Philosophy of Science*, 74, 588-600.

Hempel, C.G. and Oppenheim, P. 1948, "Studies in the Logic of Explanation", *Philosophy of Science*, 15, 2, 135-75.

Hempel, C. G. 1965, *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science*, New York: Free Press.

Kitcher, P. 1993, *The Advancement of Science*, New York: Oxford University Press.

Kuhn, T.S. 1962/1970, *The Structure of Scientific Revolutions*, Chicago: The University of Chicago Press.

Kuipers, T.A.F. 1987, "A Structuralist Approach to Truthlikeness", in Kuipers, T.A.F. (ed.), *What Is Closer-to-the-Truth?*, Amsterdam: Rodopi, 79-99.

Kuipers, T.A.F. 2000. *From Instrumentalism to Constructive Realism*. Dordrecht: Springer.

Kuipers, T.A.F. 2019, *Nomic Truth Approximation Revisited*, New York: Springer.

Laudan, L. 1977, *Progress and Its Problems*, Berkeley and Los Angeles: The University of California Press.

Laudan, L. 1981, "A Confutation of Convergent Realism", *Philosophy of Science*, 48, 19-49.

Lyons, T.D. & Vickers, P. 2021 (eds.), *Contemporary Scientific Realism: The Challenge from the History of Science*, Oxford: Oxford University Press.

Niiniluoto, I. 1987, *Truthlikeness*, Dordrecht: Reidel.

Niiniluoto, I. 1990, "Measuring the Success of Science", in *PSA 1990: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, East Lansing: The Philosophy of Science Association, 435-45.

Niiniluoto, I. 1999, *Critical Scientific Realism*, Oxford: Oxford University Press.

Niiniluoto, I. 2017, "Optimistic Realism about Scientific Progress", *Synthese*, 194, 3291-3309.

Niiniluoto, I. 2019, "Scientific Progress", in Zalta, E. (ed.), *The Stanford Encyclopedia of Philosophy*, Winter 2019 Edition.

Psillos, S. 1999, *Scientific Realism: How Science Tracks Truth*, New York: Routledge.

Putnam, H. 1975, *Mathematics, Matter and Method*, Cambridge: Cambridge University Press.

Saatsi, J. 2018 (ed.), *Routledge Handbook of Scientific Realism*, New York: Routledge.

Shan, Y. 2019, "A New Functional Approach to Scientific Progress", *Philosophy of Science*, 86, 739-58.

Smart, J.J.C. 1968, *Between Science and Philosophy*, New York: Random House.

Sprenger, J. & Hartmann, S. 2019, *Bayesian Philosophy of Science*, New York: Oxford University Press.

Stanford, P.K. 2015, "Catastrophism, Uniformitarianism, and a Scientific Realism Debate that Makes a Difference", *Philosophy of Science*, 82, 867-78.

Stanford, P.K. 2021, "Realism, Instrumentalism, Particularism: A Middle Path Forward in the Scientific Realism Debate", in Lyons, T.D. & Vickers, P. (eds.), *Contemporary Scientific Realism: The Challenge from the History of Science*, Oxford: Oxford University Press, 216-38.

van Fraassen, B. 1980, *The Scientific Image*, Oxford: Oxford University Press.

Zamora Bonilla, J. 1992, "Truthlikeness without Truth: A Methodological Approach", *Synthese*, 93, 343-72.

Zamora Bonilla, J. 1996, "Verisimilitude, Structuralism, and Scientific Progress", *Erkenntnis*, 44, 25-47.

Zamora Bonilla, J. 2000, "Truthlikeness, Rationality and Scientific Method", *Synthese*, 122, 321-35.